Discussion #2

Today: FONC, SONC, SOSC
Feasibility, Optimality Conditions

HW Due Friday Midnight
OH: T, TH 3-4

Optimization

Basic Setup:

$$\text{Min } f: \Omega \to \mathbb{R},\ \Omega \subset \mathbb{R}^n$$

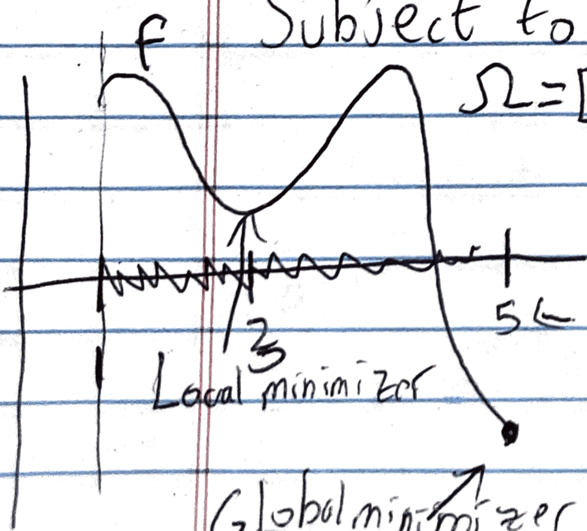
Minimize → $\min f(x)$
s.t. $x \in \Omega$

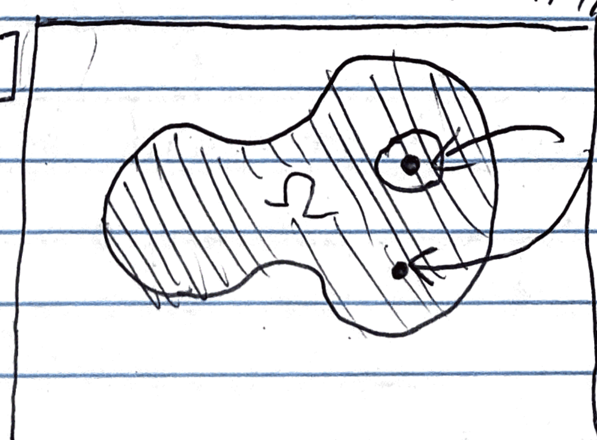
All $x \in \Omega$ are feasible points

Subject to

Domain/Feasible Set

$\Omega = [1,5]$





x^* ← minimum of f
not necessarily unique

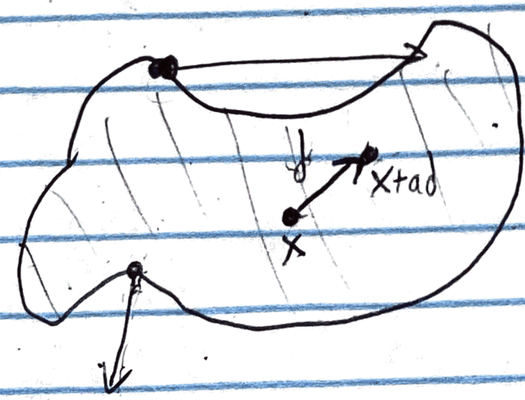
(global)

Definition: x^* is a minimizer of f on Ω if $\forall x \in \Omega$, $f(x^*) \leq f(x)$

x^* is a local minimizer if there is a nbhd B of x^* s.t. $\forall x \in B$ $f(x^*) \leq$

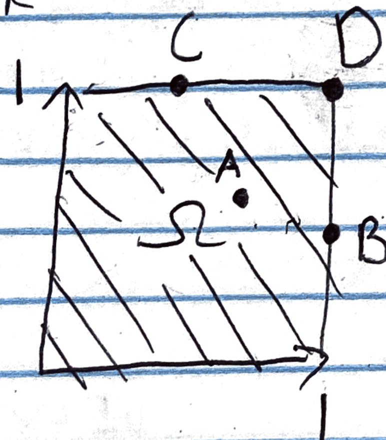
Feasible Directions:

At a point $x \in \Omega$, a vector $d \neq 0 \in \mathbb{R}^n$ is <u>feasible</u> if $\exists \alpha > 0$ s.t. for all $a \leq \alpha$, $x + ad \in \Omega$

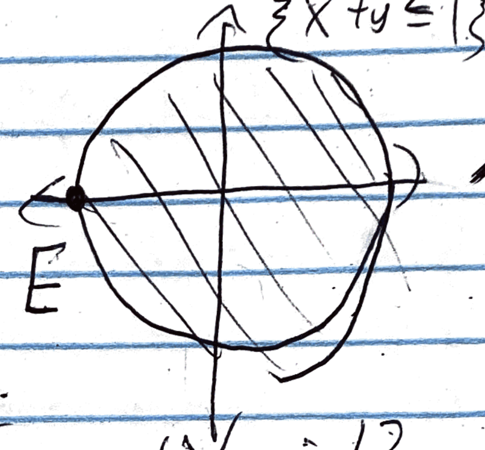


Partner work

A point is interior if all directions are feasible



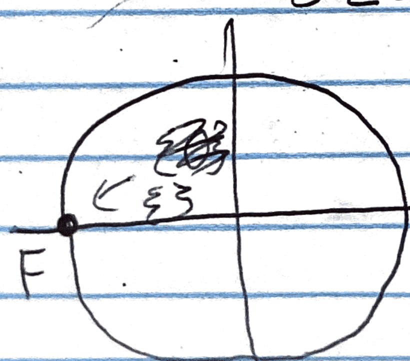
$\Omega = \{x^2 + y^2 \leq 1\}$



What are the feasible directions for each point?

A: $\mathbb{R}^2 \setminus \{0\}$
B: $\{(x,y): x \leq 0\} \setminus \{0\}$
C: $\{(x,y): y \leq 0\} \setminus \{0\}$
D: $\{(x,y): x,y \leq 0\} \setminus \{0\}$
E: $\{(x,y): x > 0\}$
F: $\{\}$

What about
$\Omega = \{x,y: x^2+y^2 =$

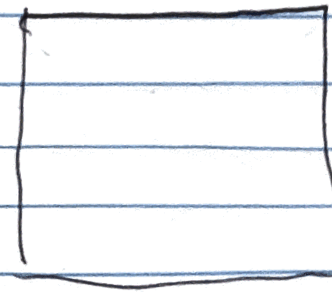


~~Basic Strategy fo~~

Review: Directional Derivative

~~what~~

Given direction d, what is derivative in direction d at point x?



$$g(a) = f(x + ad)$$

$$g'(0) = \nabla f(x) \cdot d$$

↑ Maximized when $d = \nabla f(x)$,

Hence $\nabla f(x)$ is direction of maximum increase.

FONC (Interior): $\nabla f(x) = 0$

FONC (Boundary): $d^T \nabla f(x) \geq 0$ for all feasible d

"All feasible directions are increasing"

$$f(x,y) = x + 2y$$

s.t. $(x,y) \in \Omega$, $\Omega = \begin{cases} -1 \leq x \leq 1 \\ -1 \leq y \leq 1 \end{cases}$

1. Is any point in the interior a minimizer?

2. Check the boundary