~~Thursday 12:30-2~~

Discussion #1

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This week, Wed 4-5
Th 3-4

Review of Calculus 31A, 32A
Linear Algebra

Simple function

1. $f(x) = x^2 - 4x + 3$

2. Problem: Minimize f!
Prove it is a minimum

2 steps
1. Set $f' = 0$

$$f'(x) = 2x - 4 = 0$$
$$\Rightarrow x^* = 2$$

2. Check $f''(x) > 0$

$$f''(x) = 2 > 0$$

For $x \in \text{int dom } f$

1. ~~FONC~~ First order necessary condition (FONC)

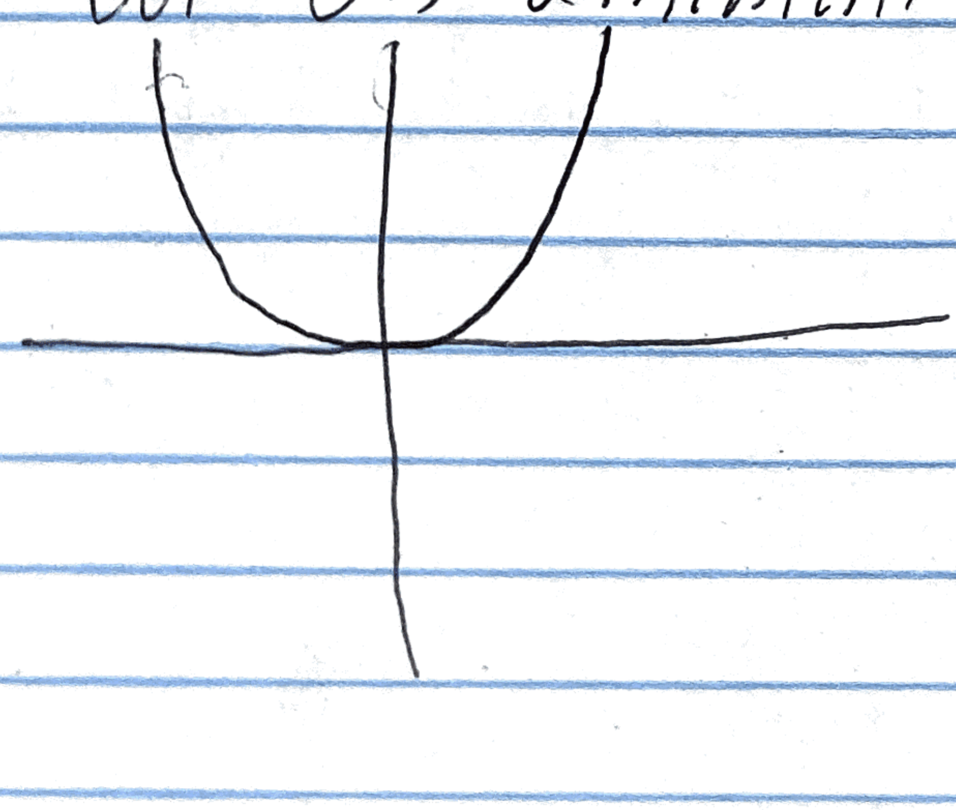
If $f'(x) \neq 0$, x is not a minimizer of f
~~(x is an interior point of domain of f)~~

2. Second order sufficient condition (SOSC)

If $f'(x) = 0$, $f''(x) > 0$,
x is a strict local minimizer of f,
i.e.,
$f(x^*) < f(y)$ for y close to x.

Not necessary to be a minimizer

$f(x) = x^4$ $f''(0) = 0$
but 0 is a minimizer



Problem: Minimize $f(x) = x^2 + 4xy + y^2 - 8x - 10y$

1. Find critical points of f?
2. Are they minimizers?

1. $\nabla f(x) = \begin{pmatrix} 2x + 4y - 8 \\ \cancel{2x}\ 4x + 2y - 10 \end{pmatrix} = 0$ (FONC)

Set $\nabla f = 0$

$2x + 4y = 8$
$-6y = -6 \Rightarrow y = 1$

$\Rightarrow$ put $y = 1$ in eq 1: $2x + 4 \cdot 1 = 8$
$\Rightarrow 2x = 4$
$x = 2$

Critical points of f is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

2. $\nabla^2 f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$

SOSC If $\nabla f(x) = 0$, $\underline{\nabla^2 f(x) > 0}$

x is a minimizer

What does this mean?

symmetric

matrix $A \in \mathbb{R}^{n \times n}$

A is positive definite (denoted $A > 0$)

1) if $\forall$ vectors $x \in \mathbb{R}^n$, $x^T A x > 0$

2) if all eigenvalues are positive

3) all principal minors are positive

E.g. $\begin{pmatrix} \boxed{1} & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

1st minor $\det \begin{pmatrix} 1 \end{pmatrix} = 1$

2nd minor $\det \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 2 - 1 = 1$

3rd minor $\det \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

$= \det \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} - \det \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} + \det \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$

$= (6 - 1) - (3 - 1) + (1 - 2)$

$= 5 - 2 - 1 = 2$

Special class of functions

If $f \in C^2 \in$ 2x cont. differentiable functions, s.t. $\nabla^2 f > 0$, it is called strictly convex

For these functions if $\nabla f = 0$, SOSC is automatically satisfied

Issues with Set $\nabla f = 0$, Check $\nabla^2 f > 0$

1. Solving $\nabla f = 0$ can be hard (maybe imposs
   (Take Numerical Analysis)
2. Solving $\nabla f = 0$ can be very expensive, (f may be a function of 1,000,000 variables)
3. ∇f may not exist
4. $\nabla^2 f$ is very expensive to calculate
   If f has a million variables
   $\nabla^2 f$ has a trillion
5. We may have domain restrictions
   e.g. minimize f
   s.t. $\cancel{(f(x)) \leq}$
   $$\|x\|_2^2 \leq 1$$