

REVERSE MATHEMATICS & NONSTANDARD ANALYSIS: WHY SOME THEOREMS ARE MORE EQUAL THAN OTHERS

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ABSTRACT. Reverse Mathematics is a program in foundations of mathematics initiated by Friedman ([1, 2]) and developed extensively by Simpson ([4]). Its aim is to determine which *minimal* axioms prove theorems of ordinary mathematics. Nonstandard Analysis plays an important role in this program ([3, 5]). We consider Reverse Mathematics where equality is replaced by the predicate \approx , i.e. equality up to infinitesimals from Nonstandard Analysis. A ‘copy’ of Reverse Mathematics for WKL_0 and ACA_0 is obtained in a weak system of Nonstandard Analysis. We consider possible connections with constructive analysis and recursion theory. Our results also have implications for the Philosophy of Science. In particular, we show how the very nature of Mathematics in Physics implies that real numbers are not needed in Physics (cf. the famous Quine-Putnam indispensability argument).

REFERENCES

- [1] Harvey Friedman, *Some systems of second order arithmetic and their use*, Proceedings of the International Congress of Mathematicians (Vancouver, B. C., 1974), Vol. 1, Canad. Math. Congress, Montreal, Que., 1975, pp. 235–242.
- [2] ———, *Systems of second order arithmetic with restricted induction, I & II (Abstracts)*, Journal of Symbolic Logic **41** (1976), 557–559.
- [3] H. Jerome Keisler, *Nonstandard arithmetic and reverse mathematics*, Bull. Symbolic Logic **12** (2006), no. 1, 100–125.
- [4] Stephen G. Simpson, *Subsystems of second order arithmetic*, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, 1999.
- [5] Kazuyuki Tanaka, *The self-embedding theorem of WKL_0 and a non-standard method*, Ann. Pure Appl. Logic **84** (1997), no. 1, 41–49.

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