Answer key of First Midterm Examination, Math 61, Version 1

1. Compute the following numbers, explain how you get the answer and write your answer in the following boxes as indicated:

a. 60	b. 62	c. 4	d. 32	e. 18

a. In a seven person committee made of persons A, B, C, D, E, F, G, in how many ways can we select a chairperson, secretary and treasurer if either A or B must be chairperson?

The number of committee choice with chair person A: $P(6, 2) = 30$,
The number of committee choice with chair person B: $P(6, 2) = 30$,
Thus the answer $= 30 + 30 = 60$.

b. How many 6-bit strings have at least one set of consecutive "00" or "11"?

6-bit strings without consecutive 00 or 11 are 101010 and 010101. Thus the total is $2^6 - 2 = 64 - 2 = 62$.

c. In the set of eight bit strings, x and y are defined to be related if first 6 bits of x and y coincide. How many elements in an equivalence classe are there?

Only the last two bit matters: The answer is $2^2 = 4$.

d. In the set of eight bit strings, x and y are defined to be related if first 5 bits of x and y coincide. How many equivalence classes are there?

Each equivalence class is determined by the first 5 bit: $2^5 = 32$ equivalence classes.

e. Two dice are rolled, one blue and one red. How many outcomes give an odd sum?

There are 3 possibilities of even outcome: 2, 4, 6 and 3 odd outcomes: 1, 3, 5. Thus the total outcome of even-odd is $3 \cdot 3 = 9$. Similraly the total outcome of odd-even is 9. Thus the answer is $9 \cdot 9 = 18$.

2. Label following statement as being true or false. In the statement below, $R \subset X \times X$ and $S \subset X \times X$ are relations, and A, B and C are subsets of a set U.

Statements	Label
$(A \cup C) - (A \cap C) = (B \cup C) - (B \cap C)$ does not necessarily implies $A = B$.	F
If R and S are anti-symmetric, then $R \circ S$ is anti-symmetric	F
If R and S are anti-symmetric, $R \cup S$ is anti-symmetric.	F
$R \cap R^{-1}$ is an equivalence relation if R is reflexive and transitive.	T
$A - (B \cup C) = (A - B) \cup C$.	F
If $\lvert A \cup B\rvert < \lvert A\rvert + \lvert B\rvert$, then $A \cap B \neq \emptyset$.	T
$A - \emptyset = \emptyset$.	F
If M is the matrix of R and N is the matrix of S, then the matrix product MN is the matrix of $R \circ S$.	F
The relation $R = \{(x, y) \lvert xy = 0\}$ is an equivalence relation on the set of all integers $\mathbb{Z}$.	F
If R and S are transitive, $R \cap S$ is transitive.	T
Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then $g \circ f$ is one to one $\Rightarrow f$ is one to one..	T
$\lvert A \cup B \cup C\rvert - \lvert A \cap B \cap C\rvert = \lvert A\rvert + \lvert B\rvert + \lvert C\rvert - \lvert A \cap B\rvert - \lvert B \cap C\rvert - \lvert C \cap A\rvert$	T
The matrix of R^{-1} is the transpose of the matrix of R.	T
$\lvert A \times (B - C)\rvert = \lvert A\rvert \times \lvert B\rvert - \lvert A\rvert \times \lvert C\rvert$.	F
If R is transitive, then R^{-1} is transitive.	T

3. Let $X = \{n : \text{integer}|13 \leq n \leq 200\}$. How many of integers in X have the digits in strictly increasing order? (123 has digits in strictly increasing order but 122 not).

Let $X_j = \{ij|10 \leq ij \leq 99, i < j\}$. Then $|X_j| = j - 1$,
Let $Y_j = \{1ij|0 \leq i, j \leq 9,\ 1 < i < j\}$. Then $|Y_j| = j - 2$.
Then we see
Ans $= \sum_{j=2}^{9} |X_j| + \sum_{j=3}^{9} |Y_j| = (2 + \cdots + 8) + (1 + 2 + \cdots + 7) = 63$.