

Section 4.6 #30. The question asks to find the number of integers between 1 and 1,000,000 that have the sum of the digits equal to 15.

This is the same as finding the number of positive integer solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 15$$

for $0 \leq x_1, \dots, x_6 \leq 9$.

We know the number of solutions for $0 \leq x_1, \dots, x_6 \leq 15 : \binom{15 + 6 - 1}{15}$.

Now we need to subtract the number of solutions in which one of the x_i 's is greater than or equal to 10, since we don't want to include cases where the integer solutions aren't single digits. So we want to solve

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 15$$

for $0 \leq x_1, \dots, x_6 \leq 15$ and $10 \leq x_i \leq 15$ for some i .

But this is the same as solving

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 15 - 10 = 5$$

for $0 \leq x_1, x_2, \dots, x_6 \leq 15$.

We know the number of solutions: $\binom{5 + 6 - 1}{5}$.

Finally, there are 6 ways of choosing the x_i we made greater than or equal to 10 above.

So the final answer is $\binom{20}{5} - 6 \cdot \binom{10}{5} = 13,992$.