Solutions to Exercises 17 and 44 of Section 1.7

Exercise 17

Basis Step:

$(1+x)^1 = 1+x = 1+1x$, so $(1+x)^1 \geq 1+1x$.

Inductive Step:

$(1+x)^{n+1} = (1+x)^n(1+x)$. By the inductive assumption, $(1+x)^n \geq 1+nx$. Since x is assumed to be ≥ -1, $1+x \geq 0$. It follows that

$$(1+x)^n(1+x) \geq (1+nx)(1+x).$$

Since $(1+nx)(1+x) = 1+(n+1)x+nx^2 \geq 1+nx$, we get that $(1+x)^{n+1} \geq 1+(n+1)x$.

Exercise 44

Note that the statement being "proved" is true for $n = 1$ and false for $n = 2$, so the argument given for the case $n = 1$ of the inductive step must be invalid.

The argument for that case begins by assuming that if a' and b' are positive integers and $\max\{a', b'\} = 1$ then $a' = b'$. (This is, of course, true.) The argument then supposes that a and b are postive integers and $\max\{a, b\} = 2$. It says that the induction hypothesis implies that $a - 1 = b - 1$. This statement is false. Either a or b might be 1. Assume, for example, that $a = 1$. Then $a-1$ is not a positive integer, so the induction hypothesis does not apply.