MATH 33A Worksheet Week 5

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Exercise 1. Find all the roots of the following polynomials: (Will need to possibly use factoring, the quadratic formula, rational roots theorem, and/or polynomial long division):

(a) $x^2 - 2x + 1$

(b) $x^2 - x - 1$

(c) $x^3 + 3x^2 - x - 3$

(d) $x^3 - 2x^2 - 2x + 4$

(Bonus: For each polynomial $p(x)$ above, can you construct a matrix A such that $\det(A - \lambda I)$)

Exercise 2. Compute the characteristic polynomial for the following matrices:

(a) $\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ with coefficients of the characteristic polynomial in terms of $\sin(\theta), \cos(\theta)$

(d) $\begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Exercise 3. For what values of $a \in \mathbb{R}$ does the following matrix have an eigenvalue of 2? $A = \begin{bmatrix} 4 & 0 & 2 \\ 2 & a & 3 \\ 0 & a^2 & 1 \end{bmatrix}$

Exercise 4. Let $A = \begin{bmatrix} 19 & -12 \\ 30 & -19 \end{bmatrix}$.

(a) What are the eigenvalues of A?

(b) Find bases for the eigenspaces of A.

(c) Using part (b), diagonalize A.

(d) Use diagonalization to find A^{100}.

Exercise 5. Diagonalize the following matrices or show that they cannot be diagonalized by showing that the geometric multiplicity of an eigenvalue is less than its algebraic multiplicity:

(a) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 9 \\ -1 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix}$ for $a \neq b \in \mathbb{R}$.

(d) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} & 0 \\ \frac{3}{5} & \frac{4}{5} & 0 \\ 1 & 2 & 2 \end{bmatrix}$

Exercise 6. True or false:

(a) If 0 is an eigenvalue of a matrix A, then $\det(A) = 0$.

(b) If a matrix only has an eigenvalue of 1, then it is hte identity matrix.

(c) All diagonalizable matrices are invertible

(d) All invertible matrices are diagonalizable
