MATH 33A Worksheet Week 4

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Exercise 1. Compute the determinant of the following matrices:

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$

Exercise 2. One important application of determinants is that they help us determine invertibility of a matrix. Use the determinant to determine for which values λ the following matrices are invertible:

(a) $\begin{bmatrix} \lambda & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & \lambda \\ 1 & \lambda & \lambda \\ \lambda & \lambda & \lambda \end{bmatrix}$

(c) $\begin{bmatrix} 1-\lambda & 2 \\ 0 & 4-\lambda \end{bmatrix}$

(d) $\begin{bmatrix} 3-\lambda & 5 & 6 \\ 0 & 4-\lambda & 1 \\ 0 & -1 & 6-\lambda \end{bmatrix}$

Exercise 3. Solve the following linear systems using Cramer's rule:

(a)

$$\begin{aligned} 3x + 4y &= 11 \\ 4x + 11y &= 3 \end{aligned}$$

(b)

$$\begin{aligned} 2x + 3y &= 8 \\ 4y + 5z &= 3 \\ 6x + 7z &= -1 \end{aligned}$$

Exercise 4. Determine whether the following are true or false:

(a) $\det(A+B) = \det(A) + \det(B)$ for any two $n \times n$ matrices A and B.

(b) If B is the rref of A, then $\det(B) = \det(A)$.

(c) There exists a 3×3 matrix A with real valued entries such that $A^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

(d) If A is an orthogonal matrix, then $\det(A) = \pm 1$.
