MATH 33A (Extra) Final Practice

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Remember to fill out course evaluations on MyUCLA!
Disclaimer: These questions may not reflect what will appear on the final.

Exercise 1. Compute the characteristic polynomial for the following matrices:

(a) $\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ with coefficients of the characteristic polynomial in terms of $\sin(\theta), \cos(\theta)$

(d) $\begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Exercise 2. For what values of $a \in \mathbb{R}$ does the following matrix have an eigenvalue of 2? $A = \begin{bmatrix} 4 & 0 & 2 \\ 2 & a & 3 \\ 0 & a^2 & 1 \end{bmatrix}$

Exercise 3. Let $A = \begin{bmatrix} 19 & -12 \\ 30 & -19 \end{bmatrix}$.

(a) What are the eigenvalues of A?

(b) Find bases for the eigenspaces of A.

(c) Using part (b), diagonalize A.

(d) Use diagonalization to find A^{100}.

Exercise 4. True or false:

(a) If V and W are dimension m and k subspaces of $\mathbb{R}^n$, then $V \cap W$ is dimension $m - k$.

(b) If A is invertible, then $\det(A^{-1}) = 1/(\det A)$.

(c) If A is an $m \times n$ matrix with $n > m$, there is a non-zero vector v such that $A \cdot v = \vec{0}$.

(d) The matrix $\begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is invertible.

(e) If A is an $m \times n$ matrix with $n < m$, there is always a solution to $Ax = \vec{b}$ for any $\vec{b} \in \mathbb{R}^n$.

(f) The rank of a matrix A is the number of leading ones in RREF.

(g) The dimension of the kernel of a matrix A is the number of columns without a leading one in RREF.

(h) If $A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ for 2 by 2 matrices A and B, then either $A = 0$ or $B = 0$.

(i) If three vectors $v_1, v_2, v_3 \in \mathbb{R}^n$ are linearly independent, then $\dim \operatorname{span}\langle v_1, v_2, v_3 \rangle = 3$.

(j) There is an orthonormal basis of $\ker A$, where $A = \begin{bmatrix} 3 & 0 & 1 \\ -6 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$.

Exercise 4. Diagonalize the following matrices or show that they cannot be diagonalized by showing that the geometric multiplicity of an eigenvalue is less than its algebraic multiplicity:

(a) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 9 \\ -1 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix}$ for $a \neq b \in \mathbb{R}$.

Exercise 5. Find a basis of the kernel and image of the matrix $A = \begin{bmatrix} 0 & -2 & 1 & 4 \\ 1 & 0 & 1 & 3 \\ -1 & -4 & 1 & 5 \end{bmatrix}$.

Exercise 6. Find the QR factorization of the following invertible matrix:

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 1 & -1 \\ -2 & 0 & 2 \end{bmatrix}$$
