# Mid term Solutions

Problem 1 :

```
w_euler(1) = w0 ;

do  i = 2, n
        w_euler(i) = w_euler(i-1) + h * deriv(t(i-1), w_euler(i-1
        temp = w_euler(i-1) + h/2 * deriv( t(i-1), w_euler(i-1));
        w_euler_double (i)        =
                temp + h/2 * deriv((t(i-1) + h/2), temp));
        error_estimate(i) = abs(w_euler(i) - w_euler_double(i)) / h;

end
```

Problem 2 :

a) Use explicit method as a predictor, implicit as a corrector

b) $dy/dt = \sin(y) \qquad y(0) = 1$

i) Predictor step :

$$w_1^{(pred)} = w_0 + h \cdot \sin\left(w_0 + \tfrac{h}{2} \sin(w_0)\right)$$

$$w_1^{(corr)} = w_0 + \tfrac{h}{2} \left( \sin(w_0) + \sin\left(w_1^{(pred)}\right)\right)$$

Problem 3 :

$$\tau(k+1) = \frac{1}{h}\left( y_{k+1} - y_k - h\, f(y_{k+1}) \right)$$

$$y_{k+1} = y_k + h y' + \frac{h^2}{2} y''$$

$$y' = \frac{dy}{dt} = f$$

$$y'' = \frac{d^2y}{dt^2} = \frac{df}{dt} = \underset{0}{\underset{\shortparallel}{\frac{\partial f}{\partial t}}} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} = f\frac{\partial f}{\partial y}$$

$$f(y_{k+1}) = f\left(y_k + h y' + \frac{h^2}{2} y''\right)$$

$$= f(y_k) + \left(h y' + \frac{h^2}{2} y''\right)\frac{\partial f}{\partial y} + \frac{1}{2}\left(h y' + \frac{h^2}{2} y''\right)^2 \frac{\partial^2 f}{\partial y^2} + \dots$$

$\Rightarrow$

$$\tau(k+1) = \frac{1}{h}\left[ y_k + h y' + \frac{h^2}{2} y'' - y_k - h f - h^2 y' \frac{\partial f}{\partial y} + O(h^3) \right]$$

$$= \frac{1}{h}\left[ \frac{h^2}{2} f \frac{\partial f}{\partial y} - h^2 y' \frac{\partial f}{\partial y} + O(h^3) \right]$$

$$= -h \frac{1}{2} f \frac{\partial f}{\partial y} + O(h^2)$$

$$\Rightarrow \quad \tau_{k+1} = O(h)$$

Problem 4:

Euler's Method : $w_{i+1} = w_i + h f(t_i, w_i)$

Model Problem: $f = \lambda w_i$

$\Rightarrow$

$w_{i+1} = w_i + h\lambda w_i$

$= (1 + h\lambda)\, w_i$

$= (1 + h\lambda)^{i+1}\, w_0$

This solution

- grows for $|1 + h\lambda| > 1$
- decays for $|1 + h\lambda| < 1$

$\Rightarrow$ decays for $h\lambda \in (-2, 0)$

b) $-50h \in (-2, 0)$

$\Rightarrow h < \frac{1}{25}$

c) $12h \notin (-2, 0)$ o.k. for all $h$