Homework 6, Problem 1

$$\frac{dy}{dt} = 0.5\, t \cos y \sin 3y \qquad 0 \le t \le 4.17$$

$$y(0) = 5$$

We want:

$$h \cdot \frac{\partial f}{\partial y} \notin R \qquad \text{for } \frac{\partial f}{\partial y} > 0$$

$$h \cdot \frac{\partial f}{\partial y} \in R \qquad \text{for } \frac{\partial f}{\partial y} < 0$$

$$f = \tfrac{1}{2} t \cancel{\sin} \cos y \sin 3y$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{1}{2} t \cdot \left[3 \cos y \cos 3y - \sin y \sin 3y \right]$$

For 4th order Runge-Kutta: $R = (-2.78, 0)$

~~for~~ $\Rightarrow$ for $\frac{\partial f}{\partial y} > 0$ $\quad h \frac{\partial f}{\partial y} \notin R$ is guaranteed

but: for $\frac{\partial f}{\partial y} < 0$:

need rough estimate: $\left[3 \cos y \cos 3y - \sin y \sin 3y \right] > -4$

$$\Rightarrow \frac{\partial f}{\partial y} > -8.34 \quad \text{for all } 0 \le t \le 4.17$$

$\Rightarrow$ timestep restriction: $-8.34\, h \in R$

$$\Rightarrow h < \frac{1}{3} \quad //$$

HW 6 Problem 3

Show that $\|\cdot\|_1$, defined as $\|x\|_1 = \sum_{i=1}^{n} |x_i|$ is norm:

i) $\|x\|_1 = \sum_{i=1}^{n} |x_i| \geq 1$, since all $|x_i| \geq 0$

ii) $\|x\|_1 = 0$, only if all $|x_i| = 0$

iii) $\|\alpha x\|_1 = \sum_{i=1}^{n} |\alpha x_i| = \sum_{i=1}^{n} |\alpha|\,|x_i| = |\alpha| \sum_{i=1}^{n} |x_i| = |\alpha|\,\|X\|_1$

iv) $\|x+y\|_1 = \sum_{i=1}^{n} |x_i + y_i| \leq \sum_{i=1}^{n} (|x_i| + |y_i|) = \|X\|_1 + \|Y\|_1$