

Homework #1

Example:

S.I.I.a) $y' = y \cos t$, $0 \leq t \leq 1$; $y(0) = 1$

1.a)

$$f(t, y) = y \cdot \cos(t)$$

$$\begin{aligned} |f(t, y_1) - f(t, y_2)| &= |y_1 \cos t - y_2 \cos t| = |y_1 - y_2| \cos t \\ &\leq 1 \cdot |y_1 - y_2| \end{aligned}$$

✓ 1.b) $y' = \frac{2}{t} y + t^2 e^t$, $1 \leq t \leq 2$; $y(1) = 0$

$$\begin{aligned} |f(t, y_1) - f(t, y_2)| &= \left| \frac{2}{t} y_1 + t^2 e^t - \frac{2}{t} y_2 - t^2 e^t \right| \\ &= \left| \frac{2}{t} (y_1 - y_2) \right| = \frac{2}{t} |y_1 - y_2| \leq 2 |y_1 - y_2| \end{aligned}$$

1.c) d) $y' = \frac{4t^3 y}{1+t^4}$, $0 \leq t \leq 1$; $y(0) = 1$

$$\begin{aligned} |f(t, y_1) - f(t, y_2)| &= \left| \frac{4t^3 y_1}{1+t^4} - \frac{4t^3 y_2}{1+t^4} \right| \\ &= \left| \frac{4t^3}{1+t^4} \right| |y_1 - y_2| \leq 2 |y_1 - y_2| \end{aligned}$$

1.b)

$$y' = g y, \quad 0 \leq t \leq t_{max}$$

$$|f(t, y_1) - f(t, y_2)| = |g y_1 - g y_2| = g |y_1 - y_2|$$

↑
Lipschitz. const.