1. Recall that an affine transformation of the plane is of the form
   \[ G(u, v) = (au + bv + c, du + ev + f). \]
   (a) Find an affine change of coordinates that takes the unit square with vertices
       \[ P = (0, 0), \quad Q = (1, 0), \quad R = (0, 1), \quad S = (1, 1) \]
       in the \( uv \)-plane to the rectangle with vertices
       \[ \bar{P} = (-1, 5), \quad \bar{Q} = (3, 5), \quad \bar{R} = (-1, 8), \quad \bar{S} = (3, 8) \]
       in the \( xy \)-plane.

   (b) Find the Jacobian for the change of coordinates in part (a).
2. Verify that
\[ f(x, y) = \begin{cases} 4xy & \text{if } 0 \leq x \leq 1, \ 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases} \]
is a joint density function. Suppose \( X \) and \( Y \) are random variables with joint density function \( f \) find \( P\left(X \geq \frac{1}{2}\right)\).

3. A spherical shell centered at the origin has an inner radius of 5 cm and an outer radius of 6 cm. The density \( \delta \) of the material increases linearly with the distance from the center. At the inner surface \( \delta = 12 \text{g/cm}^3 \) and at the outer surface \( \delta = 14 \text{g/cm}^3 \).

(a) Using spherical coordinates write the density \( \delta \) as a function of \( \rho \).

(b) Find the mass of the shell.
4. Find the Jacobian for $G : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ where $G(\rho, \theta, \phi) = (x, y, z)$ is the transformation defined by spherical coordinates.

5. Let $E$ be the ellipsoid defined by the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.$$ 

Use the change of variables $x = au$, $y = bv$, and $z = cw$ and a triple integral to find a formula for the volume of $E$ in terms of $a$, $b$, and $c$. 