Math 32B - Fall 2019
Practice Exam 2

Full Name: ________________________________

UID: ______________________________________

Circle the name of your TA and the day of your discussion:
Steven Gagniere Jason Snyder Ryan Wilkinson
Tuesday Thursday

Instructions:

• Read each problem carefully.

• Show all work clearly and circle or box your final answer where appropriate.

• Justify your answers. A correct final answer without valid reasoning will not receive credit.

• Simplify your answers as much as possible.

• Include units with your answer where applicable.

• Calculators are not allowed but you may have a 3 × 5 inch notecard.

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1. (20 points) Let $\mathbf{F}(x, y, z) = \langle e^y, xe^y, (z + 1)e^z \rangle$ and let $\mathcal{C}$ be the curve parameterized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $0 \leq t \leq 1$.

(a) Show that the vector field $\mathbf{F}$ is conservative.

(b) Find a potential function for $\mathbf{F}$.

(c) Use parts (a) and (b) to evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

(d) Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\text{curl } \mathbf{G} = \mathbf{F}$?
2. (10 points) Evaluate the line integral \( \int_C (x^2 + y^2 + z^2) \, ds \) where \( C \) is the helix parameterized by \( x = t, \ y = \cos 2t, \ z = \sin 2t \) for \( 0 \leq t \leq 2\pi \).

3. (10 points) Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F}(x, y, z) = \langle 2y + z, x - 3z, x + y \rangle \) and \( C \) is the line segment from \( (1, 0, 2) \) to \( (2, 3, -1) \).
4. (20 points) The velocity field of a fluid is given by $\mathbf{F}(x, y, z) = (x, y, z^4)$. Find the flux of the fluid across the closed surface given by $z^2 = x^2 + y^2$ for $0 \leq z \leq 1$ and $x^2 + y^2 \leq 1$ at $z = 1$ with positive orientation.
5. (20 points) Let $S$ be a portion of the helicoid parameterized by

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle \quad \text{for} \quad 0 \leq u \leq 1, \quad 0 \leq v \leq \pi.$$ 

(a) Compute $\int \int_S 2y \, dS$.

(b) Let $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$ and compute $\int \int_S \mathbf{F} \cdot d\mathbf{S}$. 
6. (10 points) Let \( \mathbf{F}(x, y) = (y^2 \cos x, x^2 + 2y \sin x) \) and let \( C \) be the path along the triangle from \((0, 0)\) to \((2, 6)\) to \((2, 0)\) and back to \((0, 0)\). Use Green’s Theorem to evaluate \( \oint_C \mathbf{F} \cdot d\mathbf{r} \).

7. (10 points) Use Green’s Theorem to find the area of the annulus \( \mathcal{R} \) bounded by two circles centered at the origin, one with radius 3 and the other with radius 5. (You should be able to check your answer easily by computing the area another way).