Math 32B - Fall 2019
Exam 2

Full Name: ________________________________

UID: ________________________________

Circle the name of your TA and the day of your discussion:
Steven Gagniere       Jason Snyder       Ryan Wilkinson
Tuesday                Thursday

Instructions:

• Read each problem carefully.

• Show all work clearly and circle or box your final answer where appropriate.

• Justify your answers. A correct final answer without valid reasoning will not receive credit.

• Simplify your answers as much as possible.

• Include units with your answer where applicable.

• Calculators are not allowed but you may have a 3 × 5 inch notecard.

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1. (20 points) Let \( F(x, y, z) = (2xy^2z, 2x^2yz, x^2y^2 + 2z) \) and let \( C \) be the line segment from \((1, 1, 3)\) to \((1, 1, -2)\).

(a) Show that the vector field \( F \) is conservative using curl.

(b) Find a function \( f \) such that \( F = \nabla f \).

(c) Use part (b) to evaluate \( \int_C F \cdot dr \).

(d) Is there a vector field \( G \) defined on \( \mathbb{R}^3 \) such that \( \text{curl} \ G = F \)?
2. (15 points) Consider the vector field

\[ \mathbf{F}(x, y) = (F_1, F_2) = \left( \frac{3}{x - y}, \frac{3}{y - x} \right). \]

(a) Show that \( \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} \).

(b) Show that \( \mathbf{F} \) is defined on two distinct connected domains in the plane. On each of these domains, is \( \mathbf{F} \) conservative? \textit{Hint}: Are these domains simply connected?

3. (15 points) Find the work done by the force field \( \mathbf{F}(x, y, z) = (x^2, y^2, z^3) \) in moving a particle along the line segment from \((0, 0, 0)\) to \((1, 2, 2)\).
4. (25 points) Let \( C \) be the curve given by the line segments from \((0, 0)\) to \((10, 0)\) to \((10, 10)\) to \((0, 10)\) as pictured below. Evaluate \( \int_C \left( e^{x^2} + 2y \right) \, dx + (5x + 2y) \, dy \). \textit{Hint:} Complete \( C \) to form a closed curve and use Green’s Theorem.
5. (25 points) Consider the vector field \( \mathbf{F}(x, y, z) = \langle x^2, y^2, 2z \rangle \) and the surface \( S \) given by \( z = xy \) for \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \). Suppose \( S \) is oriented with upward normal. Find the flux \( \iint_S \mathbf{F} \cdot d\mathbf{S} \).