Math 32B - Fall 2019
Exam 1 - V1

Full Name: ________________________________

UID: ________________________________

Circle the name of your TA and the day of your discussion:

Steven Gagniere  Jason Snyder  Ryan Wilkinson

Tuesday  Thursday

Instructions:

• Read each problem carefully.

• Show all work clearly and circle or box your final answer where appropriate.

• Justify your answers. A correct final answer without valid reasoning will not receive credit.

• Simplify your answers as much as possible.

• Include units with your answer where applicable.

• Calculators are not allowed but you may have a 3 × 5 inch notecard.

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.
1. (15 points) Evaluate the iterated integral
\[
\int_0^2 \int_{y/2}^1 \cos \left( \frac{\pi}{6} x^2 \right) \, dx \, dy.
\]

2. (10 points) Evaluate the iterated integral.
\[
\int_{-4}^4 \int_0^{\sqrt{16-x^2}} \frac{1}{\sqrt{1 + x^2 + y^2}} \, dy \, dx
\]
3. (10 points) Find a constant $C$ such that

$$p(x, y) = \begin{cases} 
Cx^2y & \text{if } 0 \leq y \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}$$

is a joint probability density function.

4. (15 points) Use a triple integral to find the volume of the solid enclosed by $x = y^2 + z^2$ and $x = 8 - y^2 - z^2$. 
5. (25 points) Let $W$ be the solid inside the sphere $x^2 + y^2 + z^2 = 4$ for $z \geq 1$. Set up but DO NOT EVALUATE a triple integral in each of the following coordinate systems that computes the mass of the solid $W$, assuming it has density function $\delta(x, y, z) = 7xy$.

1. Rectangular coordinates

2. Cylindrical coordinates

3. Spherical coordinates
6. (10 points) Use a double integral to find the area inside one loop of the polar rose 
\[ r = 3 \sin(4\theta). \] \text{Hint:} You may use the double angle formula \( \sin^2(x) = \frac{1 - \cos(2x)}{2} \).

7. (15 points) Use a change of variables to evaluate \( \int \int_{\mathcal{R}} \cos(4x^2 + 9y^2) \, dA \) where \( \mathcal{R} \) is the region in the first quadrant of the \( xy \)-plane bounded by the ellipse \( 4x^2 + 9y^2 = 1 \).