Math 32A - Winter 2019
Practice Final Exam

Full Name: ________________________________

UID: ______________________________________

Circle the name of your TA and the day of your discussion:
Qi Guo  Talon Stark  Tianqi (Tim) Wu
Tuesday  Thursday

Instructions:
• Read each problem carefully.
• Show all work clearly and circle or box your final answer where appropriate.
• Justify your answers. A correct final answer without valid reasoning will not receive credit.
• Simplify your answers as much as possible.
• Include units with your answer where applicable.
• Calculators are not allowed but you may have a 3 × 5 inch notecard.

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1. (2 points) Suppose $\mathbf{u}$ is a unit vector and $\mathbf{v}$ is a vector with $||\mathbf{v}|| = 5$. If the angle $\theta$ between $\mathbf{u}$ and $\mathbf{v}$ has $\sin \theta = \frac{3}{5}$, find the length of $\mathbf{u} \times \mathbf{v}$.

2. (3 points) Given a curve with binormal $\mathbf{B}$, show that $\frac{d\mathbf{B}}{ds}$ is perpendicular to $\mathbf{B}$.

3. (5 points) Consider the planes $3x - 2y + z = 1$ and $2x + y - 3z = 3$, which intersect in a line $L$.

   (a) Notice that the point $P = (1, 1, 0)$ is in the intersection of the planes and so is on $L$. Use $P$ to find a vector equation for $L$.

   (b) If $\theta$ is the angle between the planes, find $\cos \theta$. 

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4. (5 points) Find the equation of the plane that passes through the point (1, 2, 3) and contains the line given by the parametric equations \( x = 3t, y = 1 + t, z = 2 - t \).

5. (2 points) Suppose that \( w = f(x, y, z), y = g(s, t), \) and \( z = h(t) \). Write down the form of the chain rule you would use to compute \( \frac{\partial w}{\partial s} \) and \( \frac{\partial w}{\partial t} \).

6. (3 points) Find parametric equations for the line normal to the surface \( \sin(xyz) = x + 2y + 3z \) at the point (2, -1, 0).
7. (3 points) For what values of $x$ are the following vectors orthogonal?

$$\mathbf{v} = \langle x, x - 1, x + 1 \rangle \quad \mathbf{w} = \langle 1 - x, x + 3, 3x \rangle$$

8. (5 points) Reparametrize the following curve with respect to arc length.

$$\mathbf{r}(t) = \left( \frac{2}{t^2 + 1} - 1 \right) \mathbf{i} + \left( \frac{2t}{t^2 + 1} \right) \mathbf{j}$$
9. (5 points) The radius of a cylindrical can with top and bottom is increasing at the rate of 4 cm/sec but its total surface area remains constant at $600\pi$ cm$^2$. At what rate is the height changing when the radius is 10 cm?

10. (2 points) Show that the following function is not continuous at $(0, 0)$.

\[ f(x, y) = \begin{cases} 
\frac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\
0 & \text{if } (x, y) = (0, 0)
\end{cases} \]

11. (3 points) Show the following limit does not exist.

\[ \lim_{(x,y,z) \to (0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2} \]
12. (6 points) Let $F(x, y, z) = xy + 2xz - y^2 + z^2$.

(a) Find the directional derivative of $F(x, y, z)$ at the point $(1, -2, 1)$ in the direction of the vector $v = \langle 1, 1, 2 \rangle$.

(b) Find the maximum rate of change of $F(x, y, z)$ at the point $(1, -2, 1)$.

13. (6 points) Find and classify all critical points of the function $f(x, y) = 2x^2y - 8xy + y^2 + 5$. 
14. (12 points) Use Lagrange multipliers to find the points on the surface $x^2 + xy + y^2 + z^2 = 1$ that are closest to the origin.
15. (12 points) Let \( f(x, y) = 3 + xy - x - 2y \) and \( T \) be the closed triangular region with vertices \((1, 0), (5, 0), \) and \((1, 4)\). Find the absolute maximum and absolute minimum values of \( f \) on \( T \). Be sure to justify your answer.
16. (5 points) Find the linearization $L(x, y)$ to $f(x, y) = 1 + x \ln(xy - 5)$ at the point $(2, 3)$ and use it to approximate $f(2.01, 2.95)$.

17. (5 points) Consider the function $f(x, y, z) = z^2$ restricted to the surface $x^2 + y^2 - z = 0$. Show the method of Lagrange multipliers only gives one candidate for an extremum. Show this candidate is where $f$ has its minimum value on the surface and that $f$ has no maximum on the surface.
18. (2 points) Find and sketch the domain of the function \( f(x, y) = \sqrt{1 + x - y^2} \).

19. (2 points) For \( f(x, y) = \cos(x) - y \), sketch and label the level curves \( z = -1 \), \( z = 0 \), and \( z = 1 \).

20. (2 points) Is the following domain closed? Is it bounded?

\[ \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq z \leq 4 + x + y\} \]
21. (10 points) Consider the contour plot for $f(x, y)$ below.

(a) If a person walked from the point $(1, -1)$ to $(1, 0)$, would they be walking uphill or downhill?

(b) If a person walked from the point $(0, 0)$ to $(1, 1)$, would they be walking uphill or downhill?

(c) Is the slope steeper at $(0, -1)$ or $(2, -2)$?

(d) Is $f_y$ positive or negative at $(-1, 1)$?

(e) Determine the sign of each of the following derivatives.

\[
\begin{align*}
  f_x(1, -1) & \quad f_y(1, -1) \\
  f_{xx}(1, -1) & \quad f_{xy}(1, -1) & \quad f_{yy}(1, -1)
\end{align*}
\]

(f) Give the components of a unit vector in the direction of $\nabla f$ at the point $(-1, 1)$. (You may estimate as necessary.)