Read Section 6.7 and page 374 where positive definite and positive semidefinite operators are defined, then answer the following questions.

Let $V$ and $W$ be finite-dimensional inner product spaces and let $T: V \rightarrow W$ be a linear map of rank $r$.

1. In your own words, what are the singular values of $T$? How many singular values are there?

2. How do the singular values of $T$ relate to the singular values of $T^*$?

3. What is the pseudoinverse of $T$?

Now let $V$ be a finite-dimensional inner product space with $T: V \rightarrow V$ a self-adjoint linear operator.

4. Show that $T$ is positive definite if and only if all of its eigenvalues are positive. Furthermore, show that $T$ is positive semidefinite if and only if all of its eigenvalues are nonnegative.