Math 115B - Winter 2020
Practice Midterm Exam

Full Name: 

UID: 

Instructions:

• Read each problem carefully.

• Show all work clearly and circle or box your final answer where appropriate.

• Justify your answers. A correct final answer without valid reasoning will not receive credit.

• All work including proofs should be well organized and clearly written using complete sentences.

• You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.

• Calculators are not allowed but you may have a $3 \times 5$ inch notecard.

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.
1. (10 points) True or False: Prove or disprove the following statements.

Let $V$ be a finite-dimensional inner product space over $\mathbb{F} = \mathbb{C}$. Let $T : V \to V$ be a linear operator and $T^*$ its adjoint.

(a) The linear operator $S = T + T^*$ is diagonalizable.
(b) If $T$ is normal then $||Tv|| = ||T^*v||$ for all $v \in V$. 
2. (10 points) Let $V$ be a finite-dimensional vector space and let $T$ and $S$ be linear operators on $V$. Suppose $V$ is a $T$-cyclic subspace of itself. Show that $T$ and $U$ commute if and only if $U = g(T)$ for some polynomial $g(t)$. 


3. (10 points) Let $T : V \to V$ be a linear operator on a finite-dimensional vector space over a field $F$. Let $T^t : V^* \to V^*$ be its dual. Show that a subspace $W \subseteq V$ is $T$ invariant if and only if $W^0$ is $T^t$-invariant.
4. (10 points) True or False: Prove or disprove the following statements.

(a) Let $V$ be a finite-dimensional inner product space and let $T: V \to V$ be a linear operator. If all the eigenvalues of $T$ are 1, then $T$ must be an isometry.

(b) Let $\beta = \{1, x, x^2\}$ be the standard basis for $V = P_2(\mathbb{R})$. There exists a basis for $V$ such that the dual basis for $V^*$ is given by $\{f_0, f_1, f_2\}$ with $f_0(p(x)) = p(0)$, $f_1(p(x)) = p(1)$, and $f_2(p(x)) = p(2)$. 