All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Tuesday, March 10th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. From the book:

   Section 7.3 problems 1, 2 a, b, d, 3 b, 5

2. Let $T : V \rightarrow V$ be a linear operator on an $n$-dimensional vector space $V$. Suppose that $\dim(\ker(T^{n-1})) \neq \dim(\ker(T^n))$. Show that $\dim(\ker(T^k)) = k$ for every $0 \leq k \leq n$.

3. Let $A = \begin{pmatrix} 2 & -1 & 5 \\ 0 & 0 & -9 \\ 0 & 1 & 6 \end{pmatrix}$.

   (a) Find the eigenvalues of $A$.

   (b) Find the dimensions of the generalized eigenspaces of $A$.

   (c) Find Jordan canonical bases for the generalized eigenspaces.

   (d) Put these bases together to give a Jordan canonical basis of $\mathbb{R}^3$ for $A$ and write $A$ in Jordan canonical form.

4. Let $J_m(\lambda)$ and $J_m(\mu)$ be $m \times m$ Jordan blocks corresponding to eigenvalues $\lambda$ and $\mu$. Show that $J_m(\lambda)J_m(\mu) = J_m(\mu)J_m(\lambda)$.

5. Let $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$. Find $\lim_{n \to \infty} A^n$. 