Math 115A - Spring 2019
Practice Exam 2

Full Name: ________________________________

UID: ________________________________

Instructions:

• Read each problem carefully.

• Show all work clearly and circle or box your final answer where appropriate.

• Justify your answers. A correct final answer without valid reasoning will not receive credit.

• All work including proofs should be well organized and clearly written using complete sentences.

• You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.

• Calculators are not allowed but you may have a 3 × 5 inch notecard.

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.
1. (10 points) True or False: Prove or disprove the following statements.

(a) If $T : V \rightarrow W$ is a linear map between two $n$-dimensional vector spaces then $T$ is onto if and only if $T$ is one-to-one.

(b) If $T : V \rightarrow W$ is a linear map between two finite-dimensional vector spaces then $T$ is an isomorphism if and only if $T$ maps any basis $\beta$ for $V$ to a basis $T(\beta)$ for $W$. 

2. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the projection onto the $x$-axis along the line $y = 2x$.

(a) Give a basis for $\mathbb{R}^2$ consisting of eigenvectors for $T$ and find their corresponding eigenvalues.

(b) Find the matrix $T$ in the standard basis for $\mathbb{R}^2$. 

3. (15 points) Let $\beta = \{1, x, x^2\}$ and $\beta' = \{1 + x + x^2, x + x^2, x^2\}$ be bases of $P_2(\mathbb{R})$.

(a) Find the change of coordinate matrix from $\beta'$ to $\beta$.

(b) Find the characteristic polynomial for the matrix found in part (a).

(c) Find the change of coordinate matrix from $\beta$ to $\beta'$. 
4. (15 points) Let $V = P_3(\mathbb{R})$ and $W = M_{2 \times 2}(\mathbb{R})$. Let

$$
\beta = \{1, x, x^2, x^3\}
$$

$$
\gamma = \left\{ w_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, w_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, w_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, w_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}
$$

be the standard bases. Consider the linear map $T : V \to W$ defined by

$$
T(ax^3 + bx^2 + cx + d) = \begin{pmatrix} a + b & c + d \\ a + c & b + c \end{pmatrix}.
$$

(a) Determine $M = [T]_\beta^\gamma$.

(b) Prove that $T$ is an isomorphism.

(c) Prove that $V$ and $W$ are isomorphic without using $T$. 