Math 115A - Spring 2019
Practice Exam 1

Full Name: ________________________________

UID: ________________________________

Instructions:

• Read each problem carefully.

• Show all work clearly and circle or box your final answer where appropriate.

• Justify your answers. A correct final answer without valid reasoning will not receive credit.

• All work including proofs should be well organized and clearly written using complete sentences.

• You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.

• Calculators are not allowed but you may have a 3 × 5 inch notecard.

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.
1. (10 points) True or False: Prove or disprove the following statements.

(a) If $U_1, U_2,$ and $W$ are subspaces of a finite-dimensional vector space $V$ such that $U_1 + W = U_2 + W$, then $U_1 = U_2$.

(b) Fix an $n \times n$ matrix $B$ and let $W = \{ A \in M_{n\times n}(\mathbb{F}) \mid AB = BA \}$. Then $W$ is a subspace of $M_{n\times n}(\mathbb{F})$. 
2. (15 points) True or False: Prove or disprove the following statements.

(a) The set $W = \{(a, b, c) \in \mathbb{R}^3 \mid a^2 + b^2 + c^2 = 0\}$ is a subspace of $\mathbb{R}^3$.

(b) The set $W = \{(a, b, c) \in \mathbb{R}^3 \mid a + b + c = 0\}$ is a subspace of $\mathbb{R}^3$.

(c) There exists a linear transformation $T : \mathbb{F}^5 \to \mathbb{F}^2$ with

$$\ker T = \{(a, b, c, d, e) \in \mathbb{F}^5 \mid a = b \text{ and } c = d = e\}.$$
3. (10 points) True or False: Prove or disprove the following statements.

(a) Let \( S = \{(1, -1, 0), (0, 1, -1), (1, 1, 1)\} \subseteq \mathbb{R}^3 \). The list \( S \) is a basis for \( \mathbb{R}^3 \).

(b) Let \( B = \{(1, -1, 0), (0, 1, -1), (1, 1, 1)\} \subseteq (\mathbb{F}_2)^3 \). The list \( B \) is a basis for \( (\mathbb{F}_2)^3 \).
4. (10 points) True or False: Let \( W_1 \) and \( W_2 \) be subspaces of a vector space \( V \) over a field \( \mathbb{F} \). Prove or disprove the following sets are subspaces of \( V \).

(a) The intersection of \( W_1 \) and \( W_2 \), given by

\[
W_1 \cap W_2 = \{ v \in V \mid v \in W_1 \text{ and } v \in W_2 \}.
\]

(b) The difference of \( W_1 \) from \( W_2 \), given by

\[
W_2 - W_1 = \{ v \in V \mid v \in W_2 \text{ and } v \notin W_1 \}.
\]