Midterm Exam
APPLIED DIFFERENTIAL EQUATIONS 266B
Winter 2007

1. Solve the initial value problem

$$\begin{aligned} u_x + x u_y &= y \\ u(x,0) &= 1. \end{aligned}$$

2. Find the characteristic equations for the following nonlinear Hamilton-Jacobi equation:

$$u_t + u u_x^2 = x u^2.$$

3. For the hyperbolic system on $0 \leq x \leq 1$ and $t \geq 0$

$$\begin{aligned} u_t + u_x &= 0 \\ v_t - v_x &= 0 \\ u(x,0) &= g(x) \\ v(x,0) &= h(x) \end{aligned}$$

which of the following boundary data make the system well-posed?

(a) IBVP1

$$\begin{aligned} u(0,t) &= k(t) \\ v(1,t) &= \ell(t). \end{aligned}$$

(b) IBVP2

$$\begin{aligned} v(0,t) &= k(t) \\ u(1,t) &= \ell(t). \end{aligned}$$

(c) IBVP3

$$\begin{aligned} u(0,t)+v(0,t) &= k(t) \\ u(1,t)-v(1,t) &= \ell(t). \end{aligned}$$

4. Consider the initial value problem for the three dimensional wave equation with spherically symmetric initial data (in which $r = |\mathbf{x}|$)

$$\begin{aligned} u_{tt} &= \Delta u \quad \text{for } t > 0 \\ u(\mathbf{x},0) &= g(r) \\ u_t(\mathbf{x},0) &= h(r) \end{aligned}$$

- From the general solution formula (Kirchoff's formula), show that

$$u(0,t) = \partial_t(t\ g(t)) + t\ h(t)$$

- For the specific functions

$$\begin{aligned} h(r) &= 0 \quad \text{for all } r \\ g(r) &= \begin{cases} 0 & \text{for } r \leq 1 \\ r-1 & \text{for } 1 \leq r \end{cases} \end{aligned}$$

find a time t at which $u(0,t)$ is discontinuous. Since the initial data is continuous, this is an example of loss of regularity.