## Midterm Exam APPLIED DIFFERENTIAL EQUATIONS 266B Winter 2006

1. Solve the initial value problem

$$\begin{aligned} xu_x - yu_y &= \cos(y) \\ u(x,0) &= x. \end{aligned}$$

2. Find the characteristic equations for the following PDE:

$$u_x^2 + u_y^4 = x^2 u^2.$$

3. Solve the initial value problem

$$u_{tt} = c^2 u_{xx}$$
$$u(x,0) = e^x$$
$$u_t(x,0) = e^{2x}.$$

4. For  $at \leq x$  and  $t \geq 0$ , and for some constant a, consider

$$u_{tt} = c^2 u_{xx}$$
$$u(x,0) = g(x)$$
$$u_t(x,0) = h(x)$$
$$u(x = at, t) = k(t).$$

- Show that this is well-posed if |a| < c but ill-posed if |a| > c.
- For |a| < c, find the solution of this problem.
- 5. Consider the initial value problem

$$u_{tt} = c^2 \Delta u$$

for  $x \in \mathbb{R}^d$  and t > 0, and with  $u(x,0) = u_0(x)$   $u_t(x,0) = u_1(x)$  in which  $u_0(x) = u_1(x) = 0$  for  $|x| < \mathbb{R}_1$  and  $|x| > \mathbb{R}_2$ , with  $\mathbb{R}_1 < \mathbb{R}_2$ . For d = 2 and d = 3, find the largest set  $\Omega_0 \subset \{x \in \mathbb{R}^d, t > 0\}$  on which u = 0 for any choice of  $u_0$  and  $u_1$ .