Midterm Solutions
Math 181, Winter 2008.

1. $f = S - X e^{-r(t-T)}$

Then $f_t = -rXe^{-r(t-T)}$

$f_S = 1 \qquad f_{SS} = 0$

So $f_t + \frac{1}{2}\sigma^2 S^2 f_{SS} + rSf_S - rf = -rXe^{-r(T-t)} + rS - r\left(S - Xe^{-r(t-T)}\right)$

$= 0$

i.e. f solves Black-Scholes

The "initial conditions" are $f = S - X$ at $t = T$

2. Price for call c and put p computed here.

$X=100,\ S=100,\ T=2,\ t=0,\ \sigma=.2,\ \mu=.1,\ r=.049$

$$d_1 = \frac{\log(S/X) + (r+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$= \frac{0+(.049+.04/2)\cdot 2}{.2\cdot\sqrt{2}} = .488$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = .488 - .2\cdot\sqrt{2} = .205$$

$N(d_1) = .69$ $\qquad N(-d_1) = .31$

$N(d_2) = .58$ $\qquad N(-d_2) = .42$

$$c = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$= 100\cdot .69 - 100\,e^{-2\cdot .049}\,.58$$

$$= 69 - 52$$

$$= 17$$

$$p = Xe^{-r(T-t)}N(-d_2) - SN(-d_1)$$

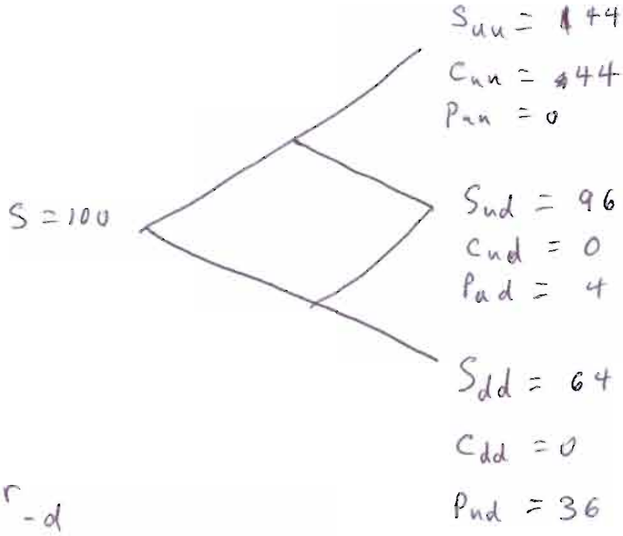
$$= 100\,e^{-2\cdot .049}\cdot .42 - 100\cdot .31$$

$$= 38 - 31$$

$$= 7$$

3. Price for put and call computed here

$u = 1.2 \quad d = .8$



$$p = \frac{e^{r} - d}{u - d} = .625$$

By risk neutral valuation

$$C_0 = e^{-2 \cdot .049}\left(.625^2 \cdot 44\right) = 15.58$$

$$P_0 = e^{-2 \cdot .049}\left(2 \cdot .625(1 - .625) \cdot 4 + (1 - .625)^2 \cdot 36\right)$$

$$= .9066\,(1.875 + 5.0625)$$

$$= 6.28$$

4. At T, $d(T) \le 1$

For cash $a(T) = 1$, $a(t) = e^{-r(T-t)}$

Since $d(T) \le a(T)$

then $d(t) \le a(t)$ by no arbitrage

So $$d(t) \le e^{-r(T-t)}$$