Math 181: Midterm Exam
February 15, 2008

1. For a forward agreement to buy a stock S for price X at time T, we found that the value is $f(S,t) = S - Xe^{-r(T-t)}$. Show that this is a solution of the Black-Scholes equation (i.e., the differential equation) and write down the "initial condition" for the price of f at time T.

2. Consider a call option on a stock S at strike price $X = 100$ at time $T = 2$ years. Suppose that the initial stock price is $S_0 = 100$, the volatility and growth rate are $\sigma = .2$ and $\mu = .1$ (both in units of years), and the risk-free rate of return is $r = 0.049$ so that $e^r = 1.05$. Find the call price $c(S_0, 0)$.

3. For the same call option, initial stock price and risk-free rate as in the previous problem, find the price $c(S_0, 0)$ based on a two step binomial random walk model for S. Let the increase and decrease factors be $u = 1.2$ and $d = 0.8$ per year, and the probability of an increase be $p' = 0.7$.

4. Consider a "digital" option d, based on an underlying stock S and a "strike price" X, whose payout at the final time T is

$$d(T) = \begin{cases} 1 & \text{if } S(T) \geq X \\ 0 & \text{if } S(T) < X. \end{cases} \tag{1}$$

   Use a no-arbitrage argument to show that the value $d(t)$ satisfies $d(t) \leq e^{-r(T-t)}$.

Table of Values of $N(x)$ and e^x.

x	$N(x)$	e^x
1.	.84	2.7
.9	.82	2.5
.8	.79	2.2
.7	.76	2.0
.6	.73	1.8
.5	.69	1.6
.4	.66	1.5
.3	.62	1.3
.2	.58	1.2
.1	.54	1.1
0.	.50	1.0
-.1	.46	.90
-.2	.42	.82
-.3	.38	.74
-.4	.34	.67
-.5	.31	.61
-.6	.27	.55
-.7	.24	.50
-.8	.21	.45
-.9	.18	.41
-1.	.16	.36