Math 181 Mortgage Backed Securities

The size of the mortgage loan pool is enormous – \$4 trillion in 1994, the largest sector of the entire debt market. Since the 1960's, mortgages are grouped together in pools and *securitized*. That means that the mortgages are used as collateral for new securities, which are called mortgage backed securities or passthrough securities. The holders of the securities receive the interest and principal payments generated by the mortgages.

We are considering here fixed rate mortgages, mainly on residential homes. Such mortgages are typically of 30 years duration. They also have prepayment possiblities. That is the borrower can pay off the loan, or any part of it at any time.

From the viewpoint of the loan issuers, which is the viewpoint of the resulting securities, the possibility of prepayment makes the security difficult to price and also presents a source of risk, called prepayment risk.

Prepayment occurs for several reasons:

– refinancing. If interest rates fall, the borrowers will pay off the high rate loan and take out a new loan at a lower rate.

– sales, caused for example by divorce, dislocation and death (the 3 D's) but also by "moving-up".

– default.

Prepayments contribute to risk if interest rates go up or down.

Down. If rates go down, then prepayments rise. But then the lender has no equivalent reinvestment possibility. This is called *contraction risk*.

Up. If rates go up, then prepayments decline. This is a loss of income. This is called extension risk.

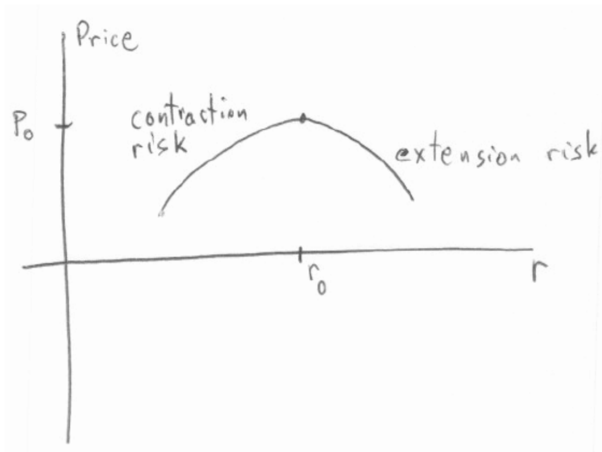

Figure 1: Value of a mortgag backed-security (MBS) as a function of interest rate r, showing that values go down for $r < r_0$ due to contraction risk and for $r > r_0$ due to extension risk.

Together the fact that values go down if either rates increase or decrease is called *negative convexity*, as shown in Figure 1.

Securitization makes it possible to bundle mortgages together to diversifying these risks. It also allows the risk to be manipulated by creating different security classes, called *tranches*, which receive interest and principal payments according to different rules. This is called a *collateralized mortgage obligation*, or CMO. These were very popular securities in the early 1990's, but in 1994 these were major losses in this market, which drove Kidder-Peabody out of business.

A CMO can consist of many different kinds of tranches. Here are some of the common types:

A: receives full interest on outstanding balance, receives all principal until completely paid off

B: receives full interest, receives all principal once A is paid off

FL: floater receives rate interest + premium e.g. 1-month LIBOR + .5%, receives principal after B

IFL: Inverse floater, receives rate that decreases as floating rate increases, e.g. 28.5% - 3 $\times$ (1 month LIBOR)

Z: accural, receives no interest until all others paid off. Instead principal increases.

Computing value of MBS

Here we omit tranches.

$$P = E\left[\sum_{k=1}^{M} u_k m_k\right] \quad (M = 360)$$

$$\begin{aligned}
u_k &= \text{discount factor} = \prod_{j=0}^{k-1}(1+i_j)^{-1} \\
i_j &= \text{monthly interest rate, month } j \\
m_k &= \text{cash flow in month } k \\
&= c r_k[(1-w_k) + w_k c_k] \\
r_k &= \text{fraction of surviving mortgages} \\
&= \prod_{1}^{k-1}(1-w_j) \\
c_k &= \text{outstanding principal} \\
&= \sum_{j=0}^{m-k}(1+i_0)^{-j} \\
c &= \text{nominal monthly payments}
\end{aligned}$$

Interest rate model

Difficult subject, but simplest model is

$$i_k = e^{\sigma\omega_k - \sigma^2/2} i_{k-1}$$

in which ω_k is $N(0,1)$. More sophisticated models include *mean reversion*, which is tendency for rates to move towards their historical average. This should be its risk-neutral rate.

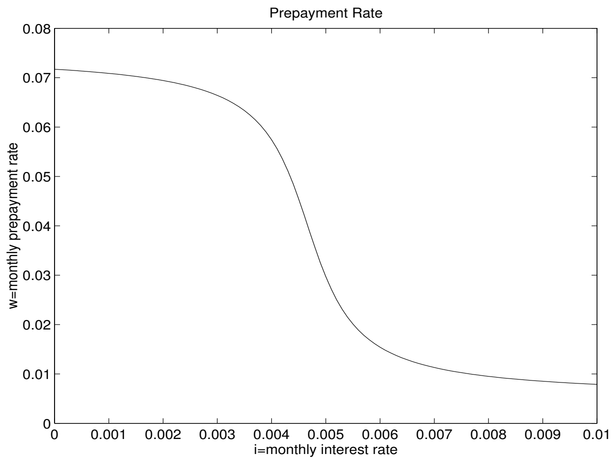

Figure 2: A simple model for prepayment rate as a function of interest rate r.

Prepayment model

Also difficult subject. Simplest model

$$\begin{aligned} w_k &= w(i_k) \\ &= K_1 + K_2 \arctan(K_3 i_k + K_4) \end{aligned}$$

as shown in Figure 2.

With i_k and w_k specified, Monte-Carlo can be used to evaluate P.

One of the most important characteristics of a mortgage or other security is its duraction

$$D = \sum_{k=1}^{M} k \frac{u_k m_k}{P}.$$

For mortgages 7 years is typical.

References

1. Collateralized Mortgage Obligations by Fabozzi, Ramirez and Ramirez

2. Mortgage-Backed Securities by Davidson and Hershovitz