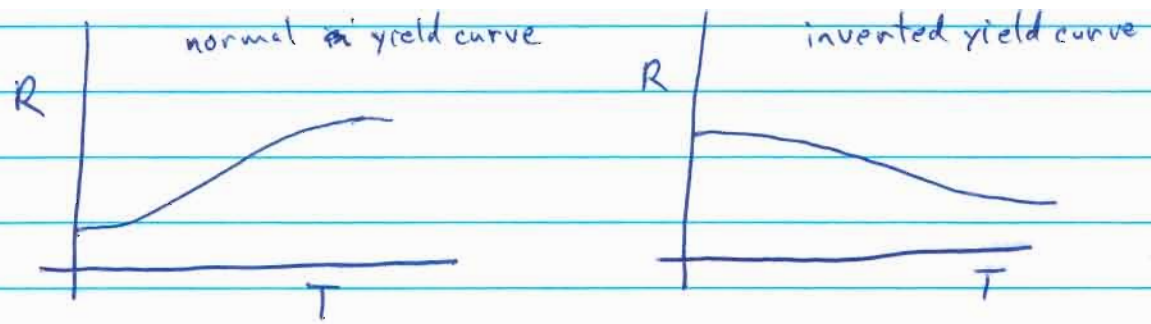
Lecture 20 Interest Rate Basics (Hull Chap 4)

There are many different rates coming from government and institutional lendors, as well as corporate ~~bonds~~ and goverment bonds. In this discussion, we will consider a single ~~rate~~ source for the interest rate, but allow for different maturities (i.e. periods of the loan) as well as changes in the current time. If the source of interest rate is a bond, we will consider only a "zero-coupon" bond, i.e. no repayments are made until the maturity (i.e. the end) of the bond period.

An interest rate R depends on the time T to maturity, i.e. $R = R(T)$, and the curve $R(T)$ vs. T is called the "term structure" or the "yield curve".



Typical yield curves, with normal or inverted behavior, are shown above.

The limit $r = R(0) = \lim_{T \to 0} R(T)$ is the "spot rate", also called the short term rate or instantaneous rate.

From $R(T)$, we can also predict future interest rates. Denote $R(0,T) = R(T)$ to be the interest rate for a period starting today ($t=0$) and ending at time $t=T$. This means that the value over the period $[0,T]$ changes by a factor $\exp(TR(0,T))$.

Now consider two maturities T_1 and T_2, with $T_2 > T_1$. Then the value changes by a factor $\exp(T_1 R(0,T_1))$ over $[0,T_1]$ and by factor $\exp(T_2 R(0,T_2))$ over $[0,T_2]$. This implies that the value changes by a factor

$$\exp(T_2 R(0,T_2)) / \exp(T_1 R(0,T_1)) = \exp((T_2 - T_1) R(T_1,T_2))$$

over the future period $[T_1, T_2]$ in which

$$R(T_1,T_2) = \frac{T_2 R(0,T_2) - T_1 R(0,T_1)}{T_2 - T_1} \qquad (20.1)$$

$$\approx R(0,T_2) + (R(0,T_2) - R(0,T_1)) \frac{T_1}{T_2 - T_1}.$$

This is called the forwarde rate over the period $[T_1, T_2]$.

In particular, if we take $T_2 \to T_1$, we get the instantaneous forward rate

$$r_f(T_1) = \lim_{T_2 \to T_1} R(T_1,T_2)$$

$$= R(0,T_1) + T_1 \frac{\partial}{\partial t} R(0,T_1) \qquad (20.2)$$

Next we discuss several types of interest rate derivatives. (Chap. 7)
An interest rate forward is an agreement to borrow funds over a future period $[T_1, T_2]$ for a specified rate $R(T_1, T_2)$.

An interest rate swap is an agreement between two parties to exchange cash flows over a future period. Most common is that one cashflow is that from a fixed interest rate, while the other cash flow is from a floating rate. Currency swaps are also common, in which the two cash flows are for principal and/or interest payments in two different currencies.

Bond options are options to buy or sell a bond (i.e. a security whose value compounds at a given interest rate) at a time T for strike price K.

An interest rate cap is an agreement in which the buyer receives money whenever the interest rate exceeds a given level (the "strike price") R_K. The amount received per unit time period is the difference $R - R_K$. In other words, the seller pays the excess interest on a loan taken out by the buyer.

An interest rate floor is the opposite: the buyer receives the difference $R_K - R$ whenever R falls below the interest rate floor R_K.

A swaption is an option to enter into an interest rate swap at some time in the future.