

Homework 1  
Math 181

1. Let  $x$  be an  $N(0, 1)$  random variable with density

$$p(x) = (2\pi)^{-\frac{1}{2}}e^{-x^2/2}.$$

Show that

- (i)  $E(x) = 0$
- (ii)  $E(x^2) = 1$
- (iii)  $E(e^{\sigma x}) = e^{\sigma^2/2}$

**Hint:** You may use the fact that  $\int_{-\infty}^{\infty} p(x)dx = 1$ . For (ii) use integration by parts. For (iii), change variables to get a difference of two squares in the exponential in the integrand.

2. Consider a random walk with steps  $d_i \pm 1$  (with probability  $1/2, 1/2$ ), as discussed in Lecture 3. Denote the values as  $x_0 = 0, x_1, x_2, \dots$ .
- (i) Find  $\text{prob}(x_4 = 4)$ .
  - (ii) Find  $\text{prob}(x_i \geq 0 \text{ for } i = 1, \dots, n)$  for  $n = 1, 2, 3, 4$ .
3. For a random walk  $x_n$  with Gaussian steps  $d_i$ , as discussed in Lecture 4, find  $\text{prob}(x_1 \geq 0)$  and  $\text{prob}(x_1 \geq 0, x_2 \geq 0)$ .