Math 181: Midterm Exam
February 7, 2007

1. Consider a European call option for a stock with initial price $S_0 = 50$, strike price $X = 70$, expiration $T = 2$ (years) and risk-free interest rate $r = .05$ (per year). Calculate the price $c(0)$ at $t = 0$ using a binary tree model of the underlying stock S, with up and down factors $u = 1.2$ and $d = .8$, time steps $dt = 1$ (years) and real probability $p' = .6$ for an up step.

2. Find the value $p(0)$ of a European put option with the same parameters as the call option in the previous problem.

3. Consider a forward agreement $F(t)$ to buy a stock $S(t)$ at strike price X and expiration time T. For the model of the stock use a binary tree with up and down factors u and d, time steps dt (years), risk-free interest rate r (per year) and real probability p' for an up step. For a one step tree with $T = dt$, show that the risk neutral valuation formula

$$F(0) = e^{-rdt}\bar{E}[F(dt)] \tag{1}$$

gives the same result as the formula

$$F(0) = S(0) - Xe^{-rT} \tag{2}$$

from the no-arbitrage argument.

4. Consider an "exotic" call option c_e with value

$$c_e(S, T) = \max(0, S + S^2 - X) \tag{3}$$

at expiration, as well as a standard call c with the same strike price X, expiration time T and underlying stock S. Use a no-arbitrage argument to show that $c_e \geq c$.