## Math 181: Midterm Exam November 3, 2003

- 1. Let S be a security with volatility  $\sigma$  and growth rate  $\mu$ , and let the risk-free rate be r. Consider an option f(S, t) with payout  $f(S, T) = S^2$  at t = T. Use the Black-Scholes PDE to find a value of  $\alpha$  such that  $f(S, t) = S^2 \exp(\alpha(T t))$ .
- 2. Suppose that f(S, t) and g(S, t) are two options with expiration at the same time T. If f(S,T) > g(S,T) for all S > 0, show that f(S,t) > g(S,t) for all S > 0 and all t with  $0 \le t \le T$ .
- 3. Consider a put option with initial price  $S_0 = 100$ , strike price X = 75, expiration T = 1.0 (years) and risk-free interest rate r = .02 (per year). Calculate the price  $p_0$  at t = 0 for two different models of the underlying stock S:
  - (a) A tree (CRR) model with up and down factors u = 1.1 and d = .9, time steps dt = .25 (years) and real probability p' = .6 for an up step.
  - (b) A Black-Scholes model with volatility  $\sigma = .4$  (in years) and  $\mu = .15$  (per year).
- 4. Let  $x_n$  be a random walk with binomial increments; i.e.,

$$\begin{array}{rcl} x_{n+1} &=& x_n \pm 1 \\ x_0 &=& 0 \end{array}$$

with equal probability of going up (+) or down (-). Find the probability that  $x_4 = 0$ .