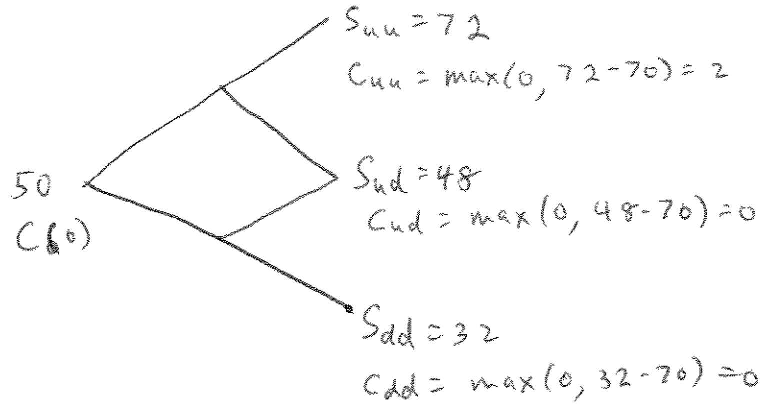
Midterm Solution Set
Math 181, Winter 2007

1. $S_0 = 50$, $X = 70$, $T = 2$

$u = 1.2$, $d = .8$, $r = .05$

The values at $t = 2$ are given in the tree below



The risk neutral probability is

$$p = \frac{e^{.05} - .8}{1.2 - .8} \cong \frac{1.05 - .8}{.4} = \frac{.25}{.4} = .625$$

So

$$\begin{aligned} c(0) &= e^{-2(.05)} E[c_2] \\ &= e^{-.1}(p^2 c_{uu} + 0) \\ &= e^{-.1}(.625)^2 \cdot 2 \\ &\cong .9 \times .39 \times 2 \\ &= .70 \end{aligned}$$

2. By put call parity

$$c - p = S - Xe^{-r(T-t)}$$

So

$$\begin{aligned} p(0) &= c(0) - S(0) + Xe^{-rT} \\ &= .70 - 50 + 70\,e^{-.1} \\ &\cong .70 - 50 + 63.3 \\ &\cong 14 \end{aligned}$$

3. The risk-neutral formula says

$$\begin{aligned} F(0) &= e^{-rdt}\,\bar{E}\,[F(dt)] \\ &= e^{-rdt}\left(p\,(uS_0 - X) + (1-p)(dS_0 - X) \right) \\ &= e^{-rdt}\left((pu + (1-p)d)\,S_0 - X \right) \end{aligned}$$

Now

$$\begin{aligned} pu + (1-p)d &= p\,(u-d) + d \\ &= \frac{e^{rdt} - d}{u-d}(u-d) + d \\ &= e^{rdt} - d + d \\ &= e^{rdt} \end{aligned}$$

So

$$\begin{aligned} F(0) &= e^{-rdt}\left(e^{rdt} S_0 - X \right) \\ &= S_0 - Xe^{-rdt} \\ &= S_0 - Xe^{-rT} \qquad \text{for } T = dt \end{aligned}$$

which agrees with the formula from no-arbitrage

4. At expiration time $t = T$

$$C_e = \max(0,\ S + S^2 - X)$$

$$c = \max(0,\ S \quad - X)$$

S_0

$$C_e(T) \geq c(T)$$

By no arbitrage

$$C_e(t) \geq c(t)$$

for all $t \leq T$.