Mathematics of Finance

Lecture 1

1.1 Course description

- modeling, analysis and numerics for pricing of financial products
- specifically *financial derivatives*; i.e. securities whose value is derived from another security or commodity
- examples: futures, options, etc.
- Black-Scholes theory – invented 1972, one of most successful math applications outside hard science
- Monte-Carlo methods – numerics for simulation of possible future events

1.2 Motivation

- financial significance
- math significance

1.3 What this course does not include

- price prediction
- portfolio strategy

- data analysis

1.4 Class info

- weekly HW
- midterm
- class project and or final

1.5 Models of equity prices

- equities (i.e. stocks) will be underlying security for most derivatives discussed here
- valuation of derivatives requires theory for evolution of stock price
- simplest theory: steady growth

$$\begin{aligned}
s_n &= \text{stock price at time} \quad t_n \\
t_n &= \text{day } n = n dt \text{ (could be weeks or months)} \\
\alpha &= \text{growth factor} \\
&= \text{fractional increase in value over some period} \\
\mu &= \alpha/dt = \text{growth rate} \\
&= \text{fractional increase/time}
\end{aligned} \tag{1}$$

The model for the evolution of stock price s_n is

$$s_{n+1} = (1+\alpha)s_n \tag{2}$$

This leads to

$$s_n = s_0(1+\alpha)^n \tag{3}$$

It is convenient to take log to get

$$\begin{aligned}
\log s_n &= \log s_0 + n\log(1+\alpha) \\
&\simeq \log s_0 + n\alpha \qquad \text{since} \log(1+\alpha) \simeq \alpha \\
&= \log s_0 + ndt\mu \\
&= \log s_0 + \mu t_n
\end{aligned} \tag{4}$$

This shows that $\log s_n$ grows linearly in $t = t_n$.

Now apply the exponential to get

$$\begin{aligned} s_n &= \exp(\log s_n) \\ &\simeq \exp(\log s_0 + \mu t_n) \\ &= s_0 e^{\mu t_n} \end{aligned} \tag{5}$$

This shows that s_n grows exponentially in $t = t_n$.

- But actual growth is *random*, not just geometric
- Next 3 lectures are on probability and stochastic processes for use in models of stock prices.