Homework 2
Math 181

1. Consider a random walk x_n with gaussian increments d_n which are $N(0,1)$ independent random variables.

   (i) Calculate $Cov(x_n, x_m) = E[x_n x_m]$ in which $m \leq n$.

   (ii) For $k < l < m < n$, show that the increments $x_l - x_k$ and $x_n - x_m$ are independent.

   (iii) Let $k < l$ and suppose that the value of x_k is known. Show that x_l is then independent of d_i for $i < k$.

2. Consider the exponential random walk model for two stocks S and S'. Assume that both have gaussian increments and that they have the same growth rate and volatility; i.e.

$$\mu = \mu' \qquad \sigma = \sigma'$$

   Also assume that the size of the time steps dt and dt' are related by

$$dt = 2dt'$$

   Let $n' = 2n$ and show that S_n and $S'_{n'}$ have the same statistics; i.e. show that they have the same probability density function.

   Hint: Show that $t_n = t'_{n'}$.