

Math 181: Midterm Exam Solution Set

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1. The payout for a call is $\max(S - K, 0)$ and for a put it is $\max(K - S, 0)$.

(a) Call payout is 10.

(b) Put payout is 0.

(c) The call is in-the-money and the put is out-of-the-money.

2.

(a) The possible values of $S(1)$ are 120 with probability $1/2$ and 80 with probability $1/2$.

(b) $E[S(1)] = .5 \times 120 + .5 \times 80 = 100$.

$$Var[S(1)] = E[(S(1) - E[S(1)])^2] = .5 \times (20)^2 + .5 \times (-20)^2 = 400..$$

(c) The possible values of $S(2)$ are $u^2 100 = 144$ with probability $p^2 = 1/4$, $du 100 = 96$ with probability $2pq = 1/2$, and $d^2 100 = 64$ with probability $q^2 = 1/4$.

3. The mean and variance for this are

$$1 = E[S(1)/S(0)] = pu + qd = .5(u + d)$$

$$1/4 = Var[S(1)/S(0)] = p(u - 1)^2 + q(d - 1)^2 = .5(u^2 + d^2 - 2(u + d) + 2) = .5(u^2 + d^2) - 2$$

from which it follows that

$$\begin{aligned} u + d &= 2 \\ u^2 + d^2 &= 5/2 \end{aligned}$$

The solution is $u = 3/2$ and $d = 1/2$.

4. The utility of the two investments is

$$\begin{aligned} U(x_1) &= .5\sqrt{4} + .5\sqrt{36} = .5(2 + 6) = 4 \\ U(x_2) &= (1/3)\sqrt{16} + (2/3)\sqrt{25} = 4/3 + 20/3 = 14/3 > 4 \end{aligned}$$

This shows that the second investment is preferable.

5. The utilities are

$$\begin{aligned} U_1(x_1) &= \log(1.1) \\ U_1(x_2) &= .5(\log(2) + \log(1/2)) = .5(\log(2) - \log(2)) = 0 < U_1(x_1) \\ U_2(x_1) &= 1.1 \\ U_2(x_2) &= .5(2 + 1/2) = 5/4 > U_2(x_1) \end{aligned}$$

This shows that the first investor favors investment 1 and the second investor favors investment 2.

6. The value of the portfolio $x(1)$ and the proportion of risky investment at time 1 are

$$\begin{aligned} x(1) &= (1+r)(x(0) - \delta(0)S(0)) + S(1) = 2(2-1) = 2 + S(1) \\ .5 &= \Pi = \delta(1)S(1)/x(1) = \delta(1)S(1)/(2 + S(1)) \end{aligned}$$

(a) If $S(1) = 3$, then $\delta(1) = 5/6$.

(b) If $S(1) = 1$, $\delta(1) = 3/2$.

(c) Case (a) involves selling and case (b) involves buying the equity.