Math 181: Midterm Exam
February 5, 2003

1. Consider an option with strike price $K = 100$ on an equity with price $S = 110$.

(a) What is the payout if the option is a call?

(b) What is the payout if the option is a put?

(c) Which of these is in-the-money and which is out-of-the-money?

2. Consider a Cox-Ross-Rubinstein (CRR) model with initial price $S(0) = 100$ which goes up by a factor $u = 1.2$ with probability $p = .5$ and down with a factor $d = .8$ with probability $q = .5$.

(a) What are the possible values of $S(1)$ and their probabilities?

(b) Compute the average $E[S(1)]$ and variance $Var[S(1)]$.

(c) What are the possible values of $S(2)$ and their probabilities?

3. Find a CRR model for which the expected return has average $E[S(1)/S(0)] = 1$ and variance $Var[S(1)/S(0)] = 1/4$, and with the additional constraint that $p = q = 1/2$.

4. Assume that your utility function is a power law $U(x) = x^{1/2}$. Consider two investments: Investment x_1 pays \$4 or \$36 each with probability 1/2. Investment x_2 pays \$16 with probability 1/3 or \$25 with probability 2/3. Which investment should you take?

5. Consider two investors, the first with a logarithmic utility function $U_1(x) = \log(x)$ (in which log denotes the natural logarithm), and the second with the utility function $U_2(x) = x$. Consider two investments, x_1 with a riskless outcome of \$1.1, and x_2 with outcome 1/2 or 2 with probability 1/2 each. Which investor will choose which investment?

6. Consider a portfolio of value x consisting of δ shares of an equity with price S and (risk-free) bonds with value $x - \delta S$. Assume that the bond price increases by a factor of $(1+r)$ in each period, that the equity follows a CRR model and that the utility is logarithmic. It follows that the proportion $\Pi = \delta S_1/x$ is a constant. Suppose that $\Pi = .5$ and $1 + r = 2$ and that $x(0) = 2$, $\delta(0) = 1$ and $S(0) = 1$.

(a) If $S(1) = 3$ find the new value $\delta(1)$.

(b) If $S(1) = 1$ find the new value $\delta(1)$.

(c) Which of these involves buying and which involves selling of the equity?