Homework 1
Math 181

Handout: Friday, Jan. 10
Due: Friday, Jan. 17

1. Let x be an $N(0,1)$ random variable with density

$$p(x) = (2\pi)^{-\frac{1}{2}} e^{-x^2/2}.$$

Show that

(i) $E(x) = 0$

(ii) $E(x^2) = 1$

(iii) $E(e^{\sigma x}) = e^{\sigma^2/2}$

Hint: You may use the fact that $\int_{-\infty}^{\infty} p(x)dx = 1$. For (ii) use integration by parts. For (iii), change variables to get a difference of two square in the exponential in the integrand.

2. Let x and y be two continuous random variables that are independent. Show that

(i) $E(x+y) = E(x) + E(y)$

(ii) $Var(x+y) = Var(x) + Var(y)$

(iii) (BONUS PROBLEM) If x and y are bivariate normal, show that $x+y$ is also normal. For simplicity take $E(x) = E(y) = 0$.