Math 181: Midterm Exam
November 9, 1998

1. Consider a forward-like security that obligates the holder to buy a stock S at a strike price X at a future time T. Note that this does not involve an exercise option; the holder is required to make the purchase. Through a no-arbitrage argument, we showed in class that the value $f(S,t)$ of this derivative security is $f(S,t) = S - Xe^{-r(T-t)}$ in which r is the risk free rate of return.

(i) Write down the Black-Scholes equation and the corresponding "initial" condition for the price $f(S,t)$, and verify that the formula above is a solution of this equation.

(ii) Show that the value of this forward security is always less than the value of the corresponding call option $c(S,t)$ with the same value of X, T, r; i.e. show that $f(S,t) < c(S,t)$. You should show this by a no-arbitrage argument.

2. Let x_n be a random walk with independent Gaussian increments ω_n which are $N(0,1)$; i.e.

$$\begin{aligned} x_{n+1} &= x_n + \omega_n \\ x_0 &= 0. \end{aligned}$$

(i) Find the variance of x_5.

(ii) Find the value of the following average: $E[\max(x_5, 0)]$.

3. Find the price p_0 at $t = 0$ for a put option with initial price $S_0 = 1.0$, strike price $X = 1.0$, expiration $T = 4.0$ and risk-free interest rate $r = 0.05$, for an underlying stock with volatility $\sigma = 0.2$.
Hint: A table of values of $N(x)$ and e^x is provided on the next page.

4. Let $p(S,t)$ be the price of a put and $c(S,t)$ be the price of a call, at the same strike price X and exercise time T, on an underlying stock with growth rate μ and volatility σ. Let r be the risk-free rate of return. As a function of t, find the stock price $S = \bar{S}$ at which the put and call have the same value; i.e. for each t, find $\bar{S}(t)$ such that $p(\bar{S}(t), t) = c(\bar{S}(t), t)$.

Table of Values of $N(x)$ and e^x.

x	$N(x)$	e^x
1.	.84	2.7
.9	.82	2.5
.8	.79	2.2
.7	.76	2.0
.6	.73	1.8
.5	.69	1.6
.4	.66	1.5
.3	.62	1.3
.2	.58	1.2
.1	.54	1.1
0.	.50	1.0
-.1	.46	.90
-.2	.42	.82
-.3	.38	.74
-.4	.34	.67
-.5	.31	.61
-.6	.27	.55
-.7	.24	.50
-.8	.21	.45
-.9	.18	.41
-1.	.16	.36