Math 181: Midterm Exam November 2, 2001

- 1. Consider the following two simple securities:
- A stock S with volatility σ and growth rate μ .
- A cash account A growing at the risk-free interest rate r.

Show that the values of these two securities each satisfy the Black-Scholes equation.

2. Consider two securities S and R both of which follow a discrete random walk, with the same factors u and d (for the up and down steps, respectively). Let p' be the (real) probability of an upstep for S and q' the (real) probability of an upstep for R. In addition, suppose that R makes an upstep whenever S makes an upstep. Show that p' = q'. (Hint: use a no-arbitrage argument over a single step of the discrete random walk.)

3. Find the price c_0 at t = 0 for a call option with initial price $S_0 = 100$, strike price X = 100, expiration T = 1.0 and risk-free interest rate $r = \ln (1.1)$ (so that $\exp r = 1.1$). Let the underlying stock be described by a tree with up and down factors u = 1.2 and d = .9, time steps dt = 1 and real probability p' = .5 for an up step.

4. Let x_n and y_m be a gaussian random walk with time step dt for x_n and δt for y_m ; i.e.

$$\begin{aligned} x_{n+1} &= x_n + \sqrt{dt}\omega_{n+1} \\ y_{m+1} &= y_m + \sqrt{\delta t}\nu_{m+1}. \end{aligned}$$

in which w_n and ν_m are all independent, standard normal random (i.e. N(0, 1)) variables. In addition suppose that $x_0 = y_0 = 0$, and that $dt = 2\delta t$. Show that x_2 and y_4 and statistically identical (i.e. they have the same probabilistic distribution).