Homework 1
Math 181

Handout: Wednesday, Oct. 1
Due: Wednesday, Oct. 8

1. Let $x$ be an $N(0,1)$ random variable with density

$$p(x) = (2\pi)^{-\frac{1}{2}} e^{-x^2/2}.$$

Show that

**(i)** $E(x) = 0$

**(ii)** $E(x^2) = 1$

**(iii)** $E(e^{\sigma x}) = e^{\sigma^2/2}$

**Hint**: You may use the fact that $\int_{-\infty}^{\infty} p(x)dx = 1$. For (ii) use integration by parts. For (iii), change variables to get a difference of two square in the exponential in the integrand.

2. Consider a random walk with steps $d_i \pm 1$ (with probability $1/2, 1/2$), as discussed in Lecture 3. Denote the values as $x_0 = 0,\ x_1,\ x_2, \ldots$.

**(i)** Find prob$(x_4 = 4)$.

**(ii)** Find prob$(x_i \geq 0$ for $i = 1, \ldots, n)$ for $n = 1, 2, 3, 4$.

3. For a random walk $x_n$ with Gaussian steps $d_i$, as discussed in Lecture 4, find prob$(x_1 \geq 0)$ and prob$(x_1 \geq 0,\ x_2 \geq 0)$.