HW#4: due Fri 5/1/2026

Problem 1: (RUDIN) EX 15, PAGE 115 (Bounding sup $|f'|$ via sup $|f|$ and sup $|f''|$)

Problem 2: (RUDIN) EX 16, PAGE 116 (Limit of f' under conditions on f and f''.)

Problem 3: (RUDIN) EX 17, PAGE 116 (Bound on f''' from 4 values of f or f'.)

Problem 4: Let $I \subseteq \mathbb{R}$ be an open interval, $n \in \mathbb{N} \smallsetminus \{0\}$ and, given $a \in I$, assume that $f\colon I \to \mathbb{R}$ (with $\mathrm{Dom}(f) = I$) is $(n-1)$-times differentiable on I and n-times differentiable at a. Prove that

$$\lim_{x \to a} \frac{f(x) - P_n(x)}{(x-a)^n} = 0$$

where $P_n(x) := \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$ is the n-th order Taylor polynomial. (This is Taylor's Theorem in asymptotic form.)

Problem 5: (RUDIN) EX 26-27, PAGE 119 (Uniqueness of ODEs via Lipschitz cont.)

Problem 6: (RUDIN) EX 15, PAGE 240 (Directional local minima w/o local minimum)

Problem 7: Let $U \subseteq \mathbb{R}^n$ be open and connected and let $f\colon \mathbb{R}^n \to \mathbb{R}^m$ with $U \subseteq \mathrm{Dom}(f)$ be differentiable on U. Prove that,

$$\big(\forall x \in U\colon\ Df(x) = 0\big) \ \Rightarrow\ f \text{ is constant}$$

Problem 8: (RUDIN) EX 27, PAGE 242 (A function with distinct mixed partials.)
