Pb Parametric curve is given by

$$x(t) = t^2 + 2t, \quad y(t) = t^3 - 3t \qquad t \in \mathbb{R}$$

Compute $\frac{d^2y}{dx^2}$ at pt. corresponding to $t=1$

Sol 1: Pretend to have $y = h(x)$ & differentiate!

$F(x,y,t) = 0$

$$F(x,y,t) = \begin{pmatrix} x - t^2 - 2t \\ y - t^3 + 3t \end{pmatrix}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

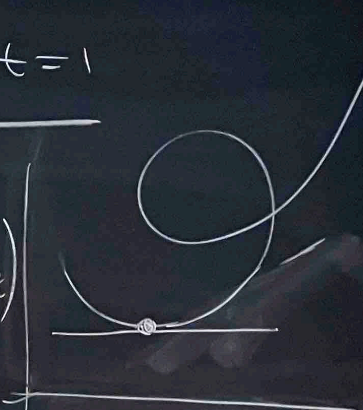
Then $h(t^2 + 2t) = t^3 - 3t$

So $h'(t^2+2t)(2t+2) = 3t^2 - 3$

$y = h(x(t))$

$dy = h'(x(t))\, x'(t)\, dt$

$h'(3) = 0$



$(x(1), y(1)) = (3, -2)$

$$h''(t^2+2t)(2t+2)^2 + \underbrace{h'(t^2+2t)}_{=0}2 = 6t$$

$$h''(3)\cdot 16 = 6 \Rightarrow h''(3) = \frac{3}{8}$$

P6.2: Assume
$$x = u + v^2$$
$$y = u^2 - v^3$$
$$z = 2uv$$

Compute $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \ldots$

$$\frac{\partial F}{\partial x} + e_1 (D_{(z,u,v)} F)\begin{pmatrix} z_x \\ u_x \\ v_x \end{pmatrix} = 0$$

$F : \mathbb{R}^5 \to \mathbb{R}^3$

$$F(x,y,z,u,v) = \begin{pmatrix} x - u - v^2 \\ y - u^2 - v^3 \\ z - 2uv \end{pmatrix}$$

WANT $F(\ldots) = 0$ solved as $z = z(x,y)$, $u = u(x,y)$, $v = v(x,y)$

$$D_{(z,u,v)} F = \begin{pmatrix} 0 & -1 & -2v \\ 0 & -2u & -3v^2 \\ 1 & -2v & -2u \end{pmatrix}$$

Unless (z ...

we get di...

Diff eqn...

1

0

$\frac{\partial z}{\partial x}$

(D...

$h'(\underbrace{t^2+2t}_{=0})2 = 6t$

$\Rightarrow h''(3) = \frac{3}{8}$

$u + v^2$

$u^2 - v^3$

$= 2uv$

$\frac{\partial f}{\partial x} + e_1 \left(D_{(z,u,v)} F\right)\begin{pmatrix} z_x \\ u_x \\ v_x \end{pmatrix} = 0$

$), \ldots$

$F : \mathbb{R}^5 \to \mathbb{R}^3$

$$F(x,y,z,u,v) = \begin{pmatrix} x - u - v^2 \\ y - u^2 - v^3 \\ z - 2uv \end{pmatrix}$$

<u>WANT</u> $F(\ldots) = 0$ solved as $z = z(x,y)$, $u = u(x,y)$, $v = v(x,y)$

$$D_{(z,u,v)} F = \begin{pmatrix} 0 & -1 & -2v \\ 0 & -2u & -3v^2 \\ 1 & -2v & -2u \end{pmatrix}$$

Unless (z,u,v) are s.t. $\det D_{(z,u,v)} F(x,y,z,u,v) = 0$ we get diff. functions $z = z(x,y)$, $v = v(x,y)$, $u = u(x,y)$.

Diff eqn's wrt x:

$$\begin{cases} 1 = \frac{\partial u}{\partial x} + 2v \frac{\partial v}{\partial x} \\ 0 = 2u \frac{\partial u}{\partial x} - 3v^2 \frac{\partial v}{\partial x} \\ \frac{\partial z}{\partial x} = 2v \frac{\partial u}{\partial x} + 2u \frac{\partial v}{\partial x} \end{cases}$$

$$\left(D_{(z,u,v)} F\right) \begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

(x,y) s.t. $\nabla f(x,y) = 0$

P3: Find critical points of f

$$f(x,y) = \frac{1}{2}y^4 - 6xy + x^2$$

and classify into local min/max or saddle pts.

$$\nabla f(x,y) = (2x-6y,\ 2y^3-6x) = 0$$

$x = 3y \wedge 2y^3 = 6x = 18y$ $\quad y^3 = 9y$

$y=0,\ y=+3,\ y=-3.$

$(0,0),\ (9,3),\ (-9,-3)$

$$\mathrm{Hess}[f](x,y) = \begin{pmatrix} 2 & -6 \\ -6 & 6y^2 \end{pmatrix}$$

at $(0,0)$ this is $\begin{pmatrix} 2 & -6 \\ -6 & 0 \end{pmatrix}$ saddle point

at $(9,3)$ this is $\begin{pmatrix} 2 & -6 \\ -6 & 54 \end{pmatrix}$ loc min

$(-9,-3)$

$$\det\begin{pmatrix} \lambda-2 & -6 \\ -6 & \lambda \end{pmatrix} = 0$$

$$\lambda(\lambda-2) = 36$$

$$\lambda^2 - 2\lambda + 1 = 37$$

$$(\lambda-1)^2 = 37$$

$$\lambda = 1 \pm \sqrt{37}$$