HW#10: due Thr 3/23/2022, 10:00AM

The purpose of this assignment is to give some practice of infinite series. (Elementary facts about these should already be familiar from Calculus.)

Problem 1: (RUDIN) PAGE 79, EX 8 ($\sum_{n=0}^{\infty} a_n b_n$ for $\{b_n\}$ monotone bounded)

Problem 2: Prove Kronecker's lemma: Supposing that $\sum_{n=0}^{\infty} a_n$ converges and $\{b_n\}_{n\geqslant 0}$ is positive, nondecreasing and unbounded, then

$$\lim_{n\to\infty} \frac{1}{b_n} \sum_{k=1}^{n} a_k b_k = 0$$

Problem 3: (RUDIN) PAGE 79, EX 9 (Radius of convergence)

Problem 4: (RUDIN) PAGE 79, EX 12 ($\sum \frac{a_n}{r_n}$ vs $\sum \frac{a_n}{\sqrt{r_n}}$ for $r_n := \sum_{k\geqslant n} a_k$)

Problem 5: (RUDIN) PAGE 80, EX 14 (Césaro averages)

Problem 6: Prove the triangle inequality for infinite sums:

$$\forall \{a_n\}_{n\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}: \quad \sum_{n=0}^{\infty} a_n \text{ convergent } \Rightarrow \left|\sum_{n=0}^{\infty} a_n\right| \leqslant \sum_{n=0}^{\infty} |a_n|$$

(The sum on the right is always meaningful in extended reals.)