Problem 1: Given two sets $A$ and $B$, consider the set $A \times B$ of ordered pairs $(x,y)$ defined as $(x,y) := \{ \{x\}, \{x,y\} \}$. Prove that
\[\forall x, \bar{x} \in A \forall y, \bar{y} \in B: \quad (x, y) = (\bar{x}, \bar{y}) \iff x = \bar{x} \land y = \bar{y} \]

Problem 2: Let $C$ be a set and consider the relation $A \subseteq B := (\forall x \in A: x \in B)$ on the powerset $\mathcal{P}(C)$ of $C$. Prove that this relation is reflexive, antisymmetric and transitive (and is thus a partial order).

Problem 3: Let $\sim$ be an equivalence relation on a set $A$ and recall that the equivalence class of $x \in A$ is defined as $[x] := \{ y \in A: x \sim y \}$
Prove that any two equivalence classes are either disjoint or equal, i.e.,
\[\forall x, y \in A: [x] = [y] \lor [x] \cap [y] = \emptyset \]

Problem 4: Let $f: X \to Y$ be a function and $\{Y_\alpha : \alpha \in I\}$ a collection of subsets of $Y$. Recall that $f^{-1}(B) := \{ x \in X: f(x) \in B \}$ for any $B \subseteq Y$. Prove the following equalities:
\[f^{-1}\left( \bigcup_{\alpha \in I} Y_\alpha \right) = \bigcup_{\alpha \in I} f^{-1}(Y_\alpha) \]
and
\[f^{-1}\left( \bigcap_{\alpha \in I} Y_\alpha \right) = \bigcap_{\alpha \in I} f^{-1}(Y_\alpha) \]

Problem 5: Ex 1.2, Page 5

Problem 6: Write $\sum_{k=1}^{n} k^4$ as a polynomial in $n$. Prove your formula by induction.

Problem 7: Ex 1.6, Page 5

Problem 8: Ex 1.8, Page 5

Problem 9: Ex 1.10, Page 6

Problem 10: Ex 1.12, Page 6