Question 1. Let \( v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \), for the following basis’ \( B \), write down the \( B \)-coordinate vector of \( v \).

(a) \( B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \)

(b) \( B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \)

Question 2. Let \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) be the linear transformation given by

\[
T(\vec{x}) = \begin{pmatrix} e_1 \cdot \vec{x} \\ e_2 \cdot \vec{x} \\ e_3 \cdot \vec{x} \end{pmatrix}
\]

where \( e_i \) are the standard basis vectors. For the following basis’ \( B \), determine the \( B \)-matrix of \( T \).

(a) The standard basis \( B = (e_1, e_2, e_3) \)

(b) The basis \( B = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \)

Question 3. Is there a basis \( B \) such that the \( B \)-matrix of \( T(\vec{x}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \vec{x} \) is upper triangular?

Question 4. (Discussion) Determine whether the following statements are true or false.

(a) There exists a \( 2 \times 2 \) matrix \( A \) such that \( \ker(A) = \text{im}(A) \).

(b) If the image of an \( n \times n \)-matrix \( A \) is all of \( \mathbb{R}^n \), then \( A \) is invertible.

(c) If a subspace \( V \) contains none of the the standard basis vectors \( e_1, \ldots, e_n \), then \( V = \{0\} \).

(d) The matrix \( I_n \) is similar to \( 2I_n \).

(e) If the matrix \( A \) is similar to \( B \), then the matrix \( A + 7I_n \) is similar to \( B + 7I_n \).

(f) If \( A^2 = 0 \) for a \( 10 \times 10 \)-matrix \( A \), then we must have that \( \text{rank} \leq 5 \).

(g) There exists a \( 2 \times 2 \)-matrix \( A \) such that \( A^2 \neq 0 \) but \( A^3 = 0 \).