Problem 1 Use geometric reasoning to find \( \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} \) and \( S \) are the following:

(a) \( \mathbf{F} = \langle 1, 0, 0 \rangle \) and \( S \) is the union of two squares \( S_1 \) and \( S_2 \) given by:
\[ S_1 : x = 0, 0 \leq y \leq 1, 0 \leq z \leq 1 \quad \text{and} \quad S_2 : z = 0, 0 \leq x \leq 1, 0 \leq y \leq 1 \]
where \( S_1 \) is oriented in the positive \( x \) direction and \( S_2 \) in the positive \( y \) direction.

(b) \( \mathbf{F} = \langle 1, 1, 0 \rangle \) and \( S \) is the square given by:
\[ S : x = 0, 0 \leq y \leq 1, 0 \leq z \leq 1 \]

(c) \( \mathbf{F} = \langle 1, 0, 0 \rangle \) and \( S \) is the cylinder given by \( x^2 + y^2 = 1 \) from \( z = 0 \) to \( z = 1 \) oriented outwards.

(d) \( \mathbf{F} = \langle x, y, 0 \rangle \) and \( S \) is the cylinder given by \( x^2 + y^2 = 4 \) and \( 1 \leq z \leq 3 \) oriented outwards.

Problem 2 Compute \( \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} = \langle xyz, xyz, xyz \rangle \) and \( S \) is the five faces of the cube \( 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1 \) missing \( z = 0 \) that is oriented outwards. \( \text{Hint: It is enough to just calculate one of the faces and multiply the result by 3. Why?} \)

Problem 3 Use green’s theorem to calculate \( \int_C \left( x^2y \, dx + (y - 3) \, dy \right) \) where \( C \) is the perimeter of the rectangle with vertices \( (1, 1), (4, 1), (4, 5) \) and \( (1, 5) \) oriented counterclockwise.

Problem 4 Compute \( \int_{\partial D} \left( \sin x - \frac{y^3}{3} \right) \, dx + \left( \sin y + \frac{x^3}{3} \right) \, dy \) where \( D \) is the annulus given in polar coordinates by \( 0 \leq \theta \leq 2\pi, 1 \leq r \leq 2 \).

Problem 5 Consider the vector field \( \mathbf{F} = \langle y, 2x \rangle \). Suppose we have two paths \( \gamma_1 \) and \( \gamma_2 \) that both start and end at the same point. How do the two line integrals of \( \mathbf{F} \) differ along the two paths?