Problem 1 Center of mass in two dimensions.

(a) Find the mass and center of mass of a triangular lamina with vertices
(0,0), (2,0), and (0,1) if the density function is \( \delta(x, y) = 1 + x + 3y \).

(b) Find the mass and center of mass of a semicircular lamina
\( x^2 + y^2 \leq R^2, x \geq 0 \) if the density function is \( \delta(x, y) = C \sqrt{x^2 + y^2} \) for some constant \( C \).
Problem 2 Center of mass in three dimensions. Suppose you are eating an ice cream cone. You determine the ice cream cone can be modeled as a solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. If the density of the ice cream cone has uniform density $\delta(x, y) = 1$, determine the mass and center of mass of the ice cream cone.
Problem 3 Probability. The joint density function for a random variables $X$ and $Y$ is

$$f(x, y) = \begin{cases} C(x + y) & \text{if } 0 \leq x \leq 4, 0 \leq y \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant $C$.

(b) Find $P(X \leq 3, Y \geq 1)$.

(c) Find $P(X + Y \leq 1)$.
Problem 4 Change of Variables.

(a) Evaluate \( \iint_R (x - 3y) \, dA \) where \( R \) is the triangular region with the vertices \((0, 0), (2, 1), (1, 2)\). Use the transformation \( x = 2u + v, y = u + 2v \).

(b) Evaluate \( \iint_R x^2 \, dA \) where \( R \) is the region bounded by the ellipse \( 9x^2 + 4y^2 = 36 \). Use the transformation \( x = 2u, y = 3v \).