1. (a) Find a change of coordinates map \( G \) that takes the unit square \([0, 1] \times [0, 1]\) to the parallelogram with vertices \((0, 0), (2, 1), (1, 2), (3, 3)\).

(b) Find the Jacobian of \( G \).

(c) Find a change of coordinates map \( G' \) that takes the unit square \([0, 1] \times [0, 1]\) to the parallelogram with vertices \((2, 1), (4, 2), (3, 3), (5, 4)\).

(d) Find the Jacobian of \( G' \) and give a geometric explanation for the similarity between the Jacobian of \( G \) and that of \( G' \).

2. Consider the region \( D \) defined by \( 1 \leq x^2 - y^2 \leq 4 \) and \( 0 \leq y \leq \frac{3x}{5} \). In this problem you’ll set up an integral to compute \( \int \int_D e^{x^2-y^2} \, dA \).

Consider the change of coordinates \( G(u, v) = \left( \frac{v}{2} + \frac{u}{2v}, \frac{v}{2} - \frac{u}{2v} \right) \). Recall from class that the inverse of this coordinate change is given by \( G^{-1}(x, y) = (x^2 - y^2, x + y) \).

(a) Find a region \( R \) of the \( uv \)-plane so that \( G : R \rightarrow D \) is a change of coordinates map (so \( G \) is onto and one-to-one on the interior of \( R \)). \textbf{Hint:} Start by finding 4 curves in the \( uv \)-plane that map to the 4 curves forming the boundary of \( D \).

(b) Give an iterated integral in \( uv \)-coordinates to compute \( \int \int_D e^{x^2-y^2} \, dA \) (No need to compute the actual integral, but it is an integral you can compute).

3. Consider the region of the part of the first quadrant \( D \) defined by \( 1 \leq x^2 + y^2 \leq 4 \) and \( 1/10 \leq xy \leq 1/2 \) and \( y \geq x \). There is a change of coordinates \( G \) that takes the rectangle \([1, 4] \times [1/10, 1/2]\) in the \( uv \)-plane to \( D \), and the inverse of this change of coordinates is given by \( G^{-1}(x, y) = (x^2 + y^2, xy) \).
(a) What is the absolute value of the Jacobian of $G^{-1}$? (It should be a function of $x$ and $y$). Pay attention to signs!

(b) Compute $\int \int_D y^2 - x^2 \, dA$

(c) Bonus problem: Note that the system of inequalities $1 \leq x^2 + y^2 \leq 4$ and $1/10 \leq xy \leq 1/2$ defines four different regions of the plane. Each of these regions can be described by a change of coordinates $G$ that takes the rectangle $[1, 4] \times [1/10, 1/2]$ in the $uv$-plane to the region where again inverse of this change of coordinates is given by $G^{-1}(x, y) = (x^2 + y^2, xy)$, but for each of these regions $G$ itself has a different formula. Find all 4 formula for $G$ and say which of these four regions goes with which formula.