1. We showed that the circle of radius $a$ has the concise description in polar coordinates as $r = a$.

Find a description of the circles $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - a)^2 = a^2$ in polar coordinates. (What if $a$ is negative?)

2. Compute the area inside the curves $r = \cos \theta$ and $r = \sin \theta$.

3. (a) Plot the curve $r = 1 + \cos(5\theta)$ (it should look something like a flower).

(b) Find the area of one “petal” of the curve $r = 1 + \cos 5\theta$. You will probably need to use the double angle formula.

4. If a solid in a region $W \subseteq \mathbb{R}^3$ has density given by $\varrho(x, y, z)$ then its mass is given by $\iiint_W \varrho(x, y, z) \, dV$.

Suppose that we are measuring in meters and consider a solid that is above the plane $z = 0$, below the paraboloid $z = 4 - (x^2 + y^2)$, and outside the cylinder $x^2 + y^2 = 1$. Suppose that the density of this solid is inversely proportional to the distance from the $z$-axis and that the density of this solid along the boundary where the paraboloid hits the $xy$-plane is $1/2 \, kg/m^3$.

Compute the mass of this solid.

5. In this problem you will find the area of the ellipse $(x/a)^2 + (y/b)^2 = 1$. We’ll use a distorted version of polar coordinates. We’ll measure points in the plane by the angle $\theta$ the line from the origin to the point makes with the $x$-axis and the value of $r$ for which the point lies on the ellipse $(x_AR)^2 + (y_{BR})^2 = 1$.

(a) Using these coordinates what point in the $xy$-plane does the value $(r, \theta)$ correspond to?

(b) Describe the ellipse $(x/a)^2 + (y/b)^2 = 1$ in these new “distorted polar coordinates”.

(c) What is the distortion factor for area with these coordinates?

For this, consider the map $(r, \theta) \mapsto (x(r, \theta), y(r, \theta))$ from the first part of the question. Differentiating this gives the matrix

$$
\begin{bmatrix}
\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\
\frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta}
\end{bmatrix},
$$

and the area distortion factor at $(r, \theta)$ is the determinant of this.
matrix, the quantity \( \frac{\partial x}{\partial r} \frac{\partial y}{\partial \theta} - \frac{\partial y}{\partial r} \frac{\partial x}{\partial \theta} \) See the picture below.

(d) What is the area enclosed by this ellipse?