Remember Stoke’s theorem: if $S$ is an oriented surface and $\partial S$ has the boundary orientation then if $\mathbf{F}$ is a vector field with continuous partial derivative then $\iint_S \nabla \times \mathbf{F} \cdot dS = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}$.

1. Let $\mathbf{F}$ be the vector field $\langle x, y, xyz \rangle$ and let $S$ be the part of the plane $2x + y + z = 2$ that lies in the first octant oriented upwards. Verify that Stoke’s theorem holds in this example by explicitly computing $\iint_S \nabla \times \mathbf{F} \cdot dS$ and $\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}$.

2. Let $S_1$ be the surface $x^2 + y^2 + 4z^2 = 4$ where $z \geq 0$ and let $S_2$ be the surface $z = 4 - x^2 - y^2$ where $z \geq 0$, where each surface is oriented with the normal pointed upwards. If $\mathbf{F}$ is a vector field with continuous partial derivatives explain why $\iint_{S_1} \nabla \times \mathbf{F} \cdot dS = \iint_{S_2} \nabla \times \mathbf{F} \cdot dS$.

3. (a) Let $D$ be the disc $x^2 + y^2 \leq 4$ with upward pointing orientation and let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle xz \sin(yz), \cos(yz), e^{x^2+y^2} \rangle$. What is $\iint_D \mathbf{F} \cdot dS$?

(b) Let $S$ be the part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$ with downward pointing orientation. What is $\iint_S \mathbf{F} \cdot dS$ (here $\mathbf{F}$ is the vector field from the previous part of the problem)? **Hint:** Does $\mathbf{F}$ have a vector potential (don’t try to explicitly find what it is!)?

4. Let $\mathcal{W}$ be the part of the solid cylinder $x^2 + y^2 \leq 1$ where $0 \leq z \leq 1$, let $\partial \mathcal{W}$ be the boundary of this solid with the outwards pointing orientation, and let $\mathbf{F} = \langle xy, yz, xz \rangle$.

(a) Directly compute $\iint_{\partial \mathcal{W}} \mathbf{F} \cdot dS$.

(b) Directly compute $\iiint_{\mathcal{W}} \text{div} \mathbf{F} \, dV$.

(c) Compare your answers– what do you notice?